

A Study of Mathematical Model of Entropy Change of Black Holes of Spin Parameters $a^*=+3/4$ & $-3/4$

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Abstract:

The present research paper discusses a mathematical model for the change in entropy of spin parameters $a^* = +3/4$ & $-3/4$ with unit angular velocity (Ω) and calculates their values in XRBs and AGN, which concludes that these black holes decrease the entropy change for co-rotation and increase the same for counter rotation and predicting the emission of radiation of Hawking radiation due to loss in mass. The interpretation of dark matter & energy is extracted from this model'

Keywords:

Hawking Radiation, XRBs and AGN

1 Introduction:

The classical theory explains only the capturing of radiation or any information near the event horizon of black, while the emission of radiation from the black holes is explained by the quantum theory (**Bardeen et al. 1973, Hawking,1974**). The black hole has a finite, non-zero temperature and entropy as predicted by Bekenstein (**Bekenstein, 2008**). A model for the changes in energy and entropy for the non-spinning black holes has been proposed (**Mahto et al. 2012**). An another model connecting the relation between the entropy and surface area given by $S=A/4$ has been also proposed by Mahto et al. which was basically postulated by Stephen Hawking (**Mahto et al. 2013**). The change in entropy of fermionic fields of black holes w.r.t. mass has been discussed (**Mahto et al. 2020**) and the same theme has been also discussed for bosonic field of black holes (**Mahto et al.2020**). Mahto et al. discussed the change in entropy of Bosonic Field of Black Holes of Spin Parameters $a^* = +2$ & -2 in XRBs and AGN (**2024**) whereas Rakesh et al also discussed the same for spin parameters $a^* = +3$ & -3 (**2024**),

The present research paper discusses a mathematical model for the change in entropy of spinning black holes w.r.t. the mass of spin parameters $a^* = +3/4$ & $-3/4$ with unit angular velocity (Ω) and calculates their values in XRBs and AGN,

2. Theoretical Discussion of Model:

The change in entropy of black holes is dependent on the spin parameter (a^*) and angular velocity (Ω) given by the following equation (Mahto & Kumari 2018).

$$\delta S / \delta M = 8\pi M (1 - 2\Omega M a^* + a^{*2} / 2 - M \Omega a^{*3}) \quad (1)$$

The model given by the equation (1) is applied for the $+3/4$ and $-3/4$ spin parameters (Yash 2022).

$$\left(\frac{\delta S}{\delta M} \right)_{+3/4} = -\frac{41\pi M}{8} (3M - 2) \quad (2)$$

$$\left(\frac{\delta S}{\delta M} \right)_{-3/4} = \frac{41\pi M}{8} (3M + 2) \quad (3)$$

The following condition must be satisfied to obtain maximum change.

$$\left(\frac{\delta S}{\delta M} \right)_{\pm 3/4} = 0 \quad (4)$$

Using the above equation into equations into (2) & (3), we have

$$M(3M - 2) = 0 \quad (5)$$

$$M(3M + 2) = 0 \quad (6)$$

The solution of eqn (5) and (6) gives

$$M = 0 \text{ or } M = 2/3 \quad (7)$$

$$M = 0 \text{ or } M = -2/3 \quad (8)$$

As per solutions, the mass of black holes may be either positive or negative or zero having their own significances.

3. Table:

Table 1: The change in entropy due to spin parameter (a^*) = $+3/4$ and $-3/4$ in XRBs.			
Sl. No	Mass (M) in ($M_\odot$)	$\left(\frac{\delta S}{\delta M}\right)_{+3/4} = -\frac{41\pi M}{8}(3M-2)$ [Joule/Kelvin/kg] $\times 10^{63}$	$\left(\frac{\delta S}{\delta M}\right)_{-3/4} = \frac{41\pi M}{8}(3M+2)$ [Joule/Kelvin/kg] $\times 10^{63}$
1	5 $M_\odot$	-4.82775	4.82775
2	6 $M_\odot$	-6.95196	6.95196
3	7 $M_\odot$	-9.46239	9.46239
4	8 $M_\odot$	-12.3590	12.3590
5	9 $M_\odot$	-15.6419	15.6419
6	10 $M_\odot$	-19.3110	19.3110
7	11 $M_\odot$	-23.3663	23.3663
8	12 $M_\odot$	-27.8078	27.8078
9	13 $M_\odot$	-32.6356	32.6356
10	14 $M_\odot$	-37.8496	37.8496
11	15 $M_\odot$	-43.4498	43.4498
12	16 $M_\odot$	-49.4362	49.4362
13	17 $M_\odot$	-55.8088	55.8088
14	18 $M_\odot$	-62.5676	62.5676
15	19 $M_\odot$	-69.7127	69.7127
16	20 $M_\odot$	-77.2440	77.2440

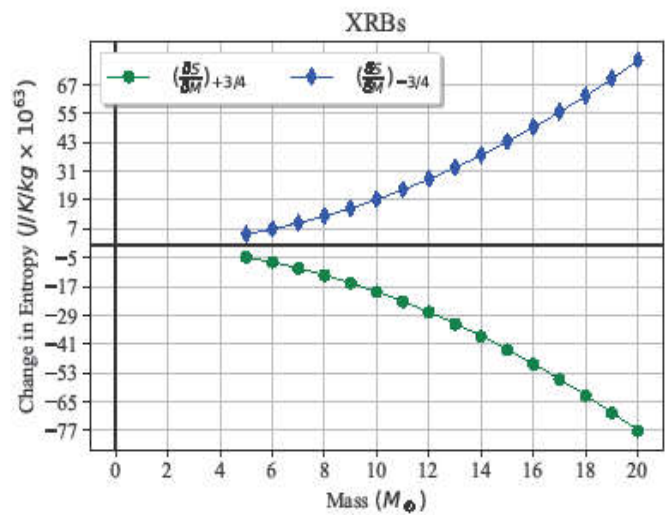
Table 2: The change in entropy due to spin parameters +3/4 and -3/4 in AGN.

S. No	Mass of BH_s (M) in solar masses	Mass of BHs in terms of $10^7 M_\odot$	$\left(\frac{\delta S}{\delta M}\right)_{+3/4} = -\frac{41\pi M}{8}(3M-2)$ [Joule/Kelvin/kg] $\times 10^{77}$	$\left(\frac{\delta S}{\delta M}\right)_{-3/4} = \frac{41\pi M}{8}(3M+2)$ [Joule/Kelvin/kg] $\times 10^{77}$
1	$1 \times 10^6 M_\odot$	.1	-0.0019311	0.0019311
2	$2 \times 10^6 M_\odot$	.2	-0.0077244	0.0077244
3	$3 \times 10^6 M_\odot$	.3	-0.0173799	0.0173799
4	$4 \times 10^6 M_\odot$	.4	-0.0308976	0.0308976
5	$5 \times 10^6 M_\odot$	.5	-0.0482775	0.0482775
6	$6 \times 10^6 M_\odot$	.6	-0.0695196	0.0695196
7	$7 \times 10^6 M_\odot$	.7	-0.0946239	0.0946239
8	$8 \times 10^6 M_\odot$	.8	-0.1235900	0.1235900
9	$9 \times 10^6 M_\odot$	.9	-0.1564100	0.1564100
10	$1 \times 10^7 M_\odot$	1	-0.1931100	0.1931100
11	$2 \times 10^7 M_\odot$	2	-0.7724400	0.7724400
12	$3 \times 10^7 M_\odot$	3	-1.7379900	1.7379900
13	$4 \times 10^7 M_\odot$	4	-3.0897600	3.0897600
14	$5 \times 10^7 M_\odot$	5	-4.8277500	4.8277500
15	$6 \times 10^7 M_\odot$	6	-6.9519600	6.9519600
16	$7 \times 10^7 M_\odot$	7	-9.4623900	9.4623900
17	$8 \times 10^7 M_\odot$	8	-12.359000	12.359000
18	$9 \times 10^7 M_\odot$	9	-15.641900	15.641900
19	$1 \times 10^8 M_\odot$	10	-19.311000	19.311000

20	$2 \times 10^8 M_{\odot}$	20	-77.244000	77.244000
21	$3 \times 10^8 M_{\odot}$	30	-173.79900	173.79900
22	$4 \times 10^8 M_{\odot}$	40	-308.97600	308.97600
23	$5 \times 10^8 M_{\odot}$	50	-482.77500	482.77500
24	$6 \times 10^8 M_{\odot}$	60	-695.19600	695.19600
25	$7 \times 10^8 M_{\odot}$	70	-946.23900	946.23900
26	$8 \times 10^8 M_{\odot}$	80	-1235.9000	1235.9000
27	$9 \times 10^8 M_{\odot}$	90	-1564.1900	1564.1900
28	$1 \times 10^9 M_{\odot}$	100	-1931.1000	1931.1000

4 Figure:

Figure 1:



The fig. 1:The change in entropy due to spin parameter +3/4 and -3/4 in XRBs.

Figure 2(a):

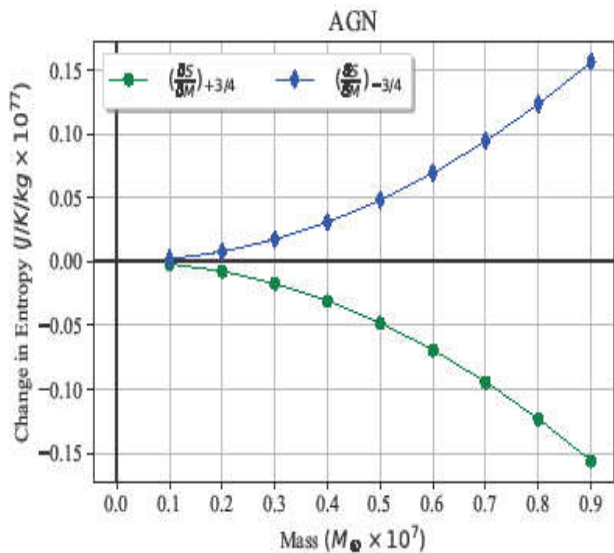


Fig. 2(a): The change in entropy of range 0.1 to 1 times $10^7 M_{\odot}$ due to spin parameter +3/4 and -3/4 in AGN.

Figure 2(b):

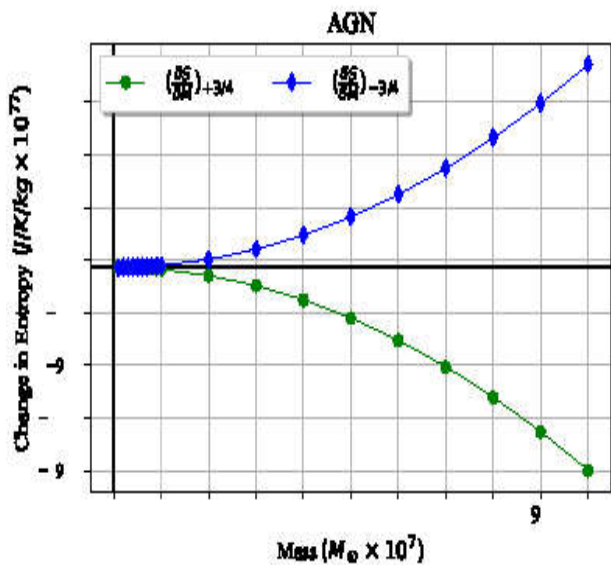


Fig. 2(b): The change in entropy of range 0.1 to 10 times $10^7 M_{\odot}$ due to spin parameter +3/4 and -3/4 in AGN.

5. Result and Discussions:

From the observations of our data calculated in the present work, we see that there are three types of masses in which one is zero and other two values are positive and negative and these values have their own significances.

The positive sign of mass gives the general concept of mass as proposed in Newtonian mechanics well known for the concept of gravity and hence justifies the laws of gravity.

The negative sign of mass gives the concept of negative mass and it leads the idea of naked singularity of black holes as explained by Schwarzschild in 1916 while solving the Einstein field equation. The negative mass also leads the concept of dark energy and dark matter.

The zero mass of any object is explained as per classical theory that the body has zero momentum and zero energy and will be remain at the rest and also exerts no force on other objects. Hence there is no existence of zero mass in nature, but Einstein was not agreed to this ideas and said that the light particle is massless and it has energy and quantum theory is postulated as radiation light consists of large number of packets of energy, each packet called quanta or photon has energy $h\nu$, where h is Planck constant and ν is the frequency of radiation. The mass of photon is also zero. The general theory of relativity also contradicts the concept of zero mass as proposed by classical theory that the light bends passing near the strong gravitational fields like black holes. This means that a body which have strong gravity exerts a force on zero mass particles.

6, Conclusions:

1. The prediction the emission of energy of Hawking radiation on the basis of quantum mechanical fields is explained
2. The interpretation of dark matter & energy is extracted from this model'

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