

**Manuscript Title:** Crack Detection in Different Dielectric Materials using Opt- shrink Rank Optimization and Statistical Thresholding.

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# Crack Detection in Different Dielectric Materials using Opt- shrink Rank Optimization and Statistical Thresholding

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**Abstract**— Crack detection in different dielectric materials using millimeter wave imaging is becoming a promising technique due to its penetration ability through the opaque materials. The most used technique for crack detection in literature is edge detection. In general, this method can suffer from false alarms if the clutter and noise is present in the received signal. Hence, adaptive image enhancement can be a potential candidate to find out the smallest crack in the materials. *Opt- shrink* algorithm based on estimating optimum rank used previously for de- noising is explored in this paper for thresholding. Moreover, this paper tries to explore probable application of millimeter wave based non- destructive crack detection using V- band (61 GHz) radar system. The performance of the proposed approach is very encouraging for the detection of cracks in low as well as high dielectric materials.

**Index Terms**— Millimeter wave imaging, crack detection, de-noising, rank optimization, thresholding.

## I. INTRODUCTION

QUALITY monitoring and testing is an essential activity in an industry for the maintenance and saving the life of the workers [1]. Mechanical and civil industries have metal ( $\epsilon_r \approx \infty$ ) and ceramic tiles ( $\epsilon_r \approx 28$ ) as an integral part of any structure. Rusting or pitting of the metallic structure is a natural phenomenon [2] while, cracks in the ceramic tiles structure is possible due to various reasons. The change in the permittivity value can be sensed by Electromagnetic (EM) signals very effectively [3]. Millimeter Wave (MMW) imaging from EM spectrum range 30- 300 GHz can be suitable for detecting any crack or damage in the structure [4]. The penetration ability of the MMW signal with better image resolution makes it suitable candidate for Non-Destructive Technique (NDT) [5]. Many researchers in MMW imaging are exploring this application for security and hidden object detection [6]. Traditionally many NDT techniques are available [7] but facing challenges related to image enhancement [2, 17].

The images developed using MMW imaging technique facing challenges due to the signal distortion and additive noise in the case of reflections from the low dielectric targets [9].

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Henceforth, it becomes necessary to de- noising the data collected at the receiver. *Opt- Shrink* algorithm proposed in [10] is a best approximation for the low rank signal matrix corrupted by the noise. Intensity based edge detection techniques such as Canny edge detection are popular among the researchers working in crack detection [11]. In [12] Canny edge detection method and Bayesian thresholding is used for the crack detection, but it has drawbacks like threshold value is not suitable for the rough and uneven surfaces. In literature very few studies are available for crack detection using MMW imaging technique. Some of the most relevant studies tried to refer here as follows: signal processing technique like compressive sensing is used to reduce the time and cost for the scanning [13]. The Percolation method is used in [14] to reduce the computation cost. The shade correction method is used in [15] for the detection of the cracks on the concrete surface, but this method suffers from higher false alarm rate. Many intensity-based thresholding techniques are available [4 - 7] but do not give satisfactory results if threshold is set below the average value [16]. S. Agarwal et. al. [2] have proposed adaptive thresholding approach for the crack detection in ceramic tiles. In [17] image morphing technique is implemented for crack detection using MMW NDT method. The objective of this paper is to develop an adaptive and novel approach which can be suitable for the detection of cracks in low as well as high dielectric materials.

In this paper, a novel approach to estimate the effective rank in *Opt- Shrink* algorithm by calculating relative mean square error (MSE) for contrast (i.e. materials having different dielectrics) targets is proposed. Finally, we use the statistical information of the data to find out the adaptive threshold value for high as well as low dielectric targets. This paper is organized into the following sections, the experimental set- up and data collection is described in section II. A novel approach to find the optimum value for the rank in *Opt- shrink* algorithm is described in section III. The proposed algorithm using Genetic Algorithm (GA) framework and statistical thresholding is described in Section IV. Finally, section V concludes the paper.

## II. EXPERIMENTAL SET- UP AND DATA COLLECTION

Indigenously assembled MMW radar system is used for the data collection. The overall MMW imaging system consists of a stepped frequency continuous wave (SFCW) radar and display.

Targets for scanning were placed on the 2-D wooden frame and slides over the 2- D structure as shown in Fig. 1 in the direction of arrows.

A. Set-up

In our experimental work metallic and ceramic tile sheets of different sizes and shapes are used for scanning in which grooves of different sizes and shapes are made to give a feel like crack in the material and covered with cardboard. Some non – faulty sheets have also been selected to test the developed approach and all these sheets are called target. Three types of scanning, namely A- scan, B- scan and C- scan are most common in MMW imaging. First the targets are placed on the wooden frame as shown in Fig.1. The standoff distance between antenna and the target is maintained as  $R = 50 \text{ cm}$ . The frequency range between  $60\text{-}62 \text{ GHz}$  is chosen for millimeter wave imaging hence the bandwidth of  $2 \text{ GHz}$ . For these specifications the calculated cross-range resolution is  $\Delta CR = \lambda R/D = 4.17 \text{ mm}$ . and down range resolution is  $\Delta R = C/2 B.W. = 7.5 \text{ cm}$ .

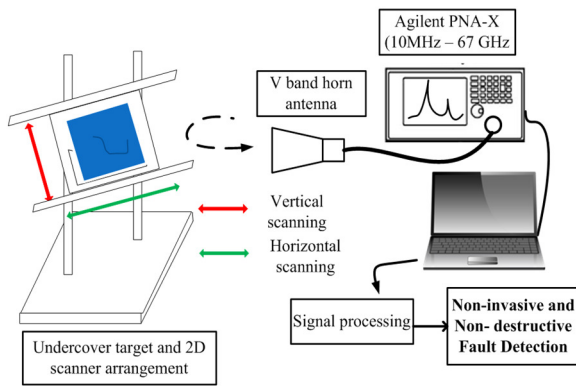


Figure1. 2-D structural setup

The overall methodology used to scan the targets is like our previous work described in [17]. The targets are selected from low and high dielectric categories so the developed method can be generalized to all types of materials. Sufficient care has been taken during scanning that targets will move smoothly on a 2-D wooden frame. Finally, c- scan raw image is developed using stacking of B- scan [18]. The developed raw images are shown in Fig. 2.

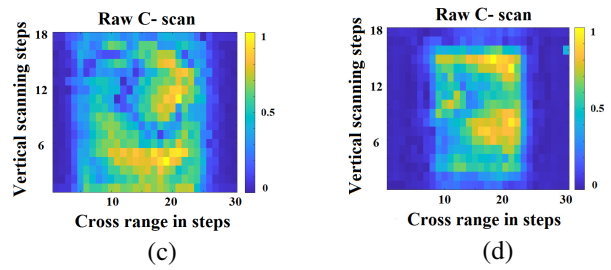
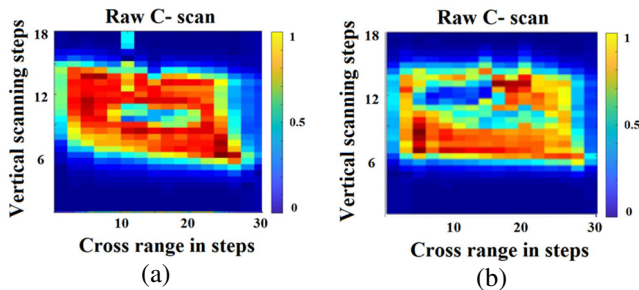


Figure2. Raw C- scan images using stacking (a) and (b) for metal targets (c) and (d) for ceramic tiles targets

III. OPT- SHRINK ALGORITHM FOR DE- NOISING

The data collected using the procedure described in section I is generally corrupted due to noise. The noise is assumed to be i.i.d. Gaussian [10], then to reduce the noise Singular Value Decomposition (SVD) can be used from LRA [19]. The best approximation for the rank from this LRA technique is truncated SVD. Recently optimum re-weighting for the rank is proposed in [10] by developing the *Opt- shrink* algorithm to recover the signal buried in noise. The basic principle from random matrix theory is used in *Opt- shrink*, where  $\tilde{X}$  is the signal received at the transmitter corrupted with the noise ( $X_n$ ) when  $S_n$  is the transmitted signal. In [10] constrained optimization problem is defined for SVD. In SVD  $S_n$  is recovered from  $\tilde{X}$  by  $S_n = \sum_{i=1}^r \theta_i u_i v_i^H$  where  $u_i$  and  $v_i$  are the unitary matrices.  $\theta_i$  are the singular values of the matrix ( $S_n$ ). The general solution for low rank matrices can be obtained using Eckart-Young-Mirsky (EYM) estimator. The objective of EYM estimator is to identify the low rank from known rank  $R$  for the matrix. The overview for *Opt- shrink* algorithm is as follows.

Consider a signal plus noise matrix.

$$\tilde{X} = S_n + X_n \tag{1}$$

Where,  $\tilde{X}$  and  $S_n$  is the received and transmitted signal respectively and  $X_n$  is the random noise. The solution for this constrained optimization problem is given by  $\widehat{S_{eym}} = \arg \min_{rank(S)=R} \|\tilde{X} - S_n\|_F$  using Frobenius norm  $\|\cdot\|_F$  and  $S_{eym} = \sum_{i=1}^r \sigma_i u_i v_i^H$  is the SVD of the received signal  $\tilde{X}$ . It is also a maximum likelihood (ML) estimation. If  $X_n$  is consider as an i.i.d. process following Gaussian distribution, then natural extension for  $S_{eym}$  will be available when  $S_n$  having low rank and sparse [20]. If the EYM estimator is further exploitable apart from low rank, then its estimation can be improved further. If  $S_n$  is isotopically random then the condition of randomness for  $X_n$  also get diluted. In this scenario *Opt- shrink* will be available for Gaussian as well as non- Gaussian cases, but for this relaxation  $S_n$  and  $X_n$  should be independent [10]. If  $X_n$  is *bi-orthogonally invariant*, then the distribution of the  $X_n$  becomes invariant after multiplying the right (or left) singular vector. The analog form of the Fourier transform is known as D- transform. D- transform is used effectively in *opt- shrink* to find optimum rank  $R$  in the following manner [10].

- 1) Decide the effective rank for SVD matrix.
- 2) Evaluate SVD for  $R$  by  $\hat{R} = \sum_{i=1}^{\hat{r}} \hat{\sigma}_i \hat{u}_i \hat{v}_i^T$
- 3) Compute  $\hat{\Sigma}_{\hat{r}} = \text{diag}(\hat{\sigma}_{\hat{r}+1} \dots \hat{\sigma}_q) \in R^{(n-\hat{r}) \times (m-\hat{r})}$
- 4) Compute  $D$ -transform for  $\hat{D}(\hat{\sigma}_i, \hat{\Sigma}_{\hat{r}})$  and  $\hat{D}'(\hat{\sigma}_i, \hat{\Sigma}_{\hat{r}})$ 

$$\hat{D}(z, x) = 1/n \text{Tr}(z(z^2 I - xx^H)^{-1}). 1/m \text{Tr}(z(z^2 I - x^H x)^{-1})$$

$$\hat{D}'(z, x) = 1/n \text{Tr}((z(z^2 I - xx^H)^{-1}). 1/m \text{Tr}(-2z(z^2 I - x^H x)^{-2}) + (z^2 I - x^H x)^{-1})$$

$$+ 1/m \text{Tr}(z(z^2 I - x^H x)^{-1}). 1/n \text{Tr}(-2z^2(z^2 I - xx^H)^{-2}) + (z^2 I - x^H x)^{-1}$$
- 5) Compute  $\omega_{i, \hat{r}}^{\text{opt}} = -2 \hat{D}(\hat{\sigma}_i, \hat{\Sigma}_{\hat{r}}) / \hat{D}'(\hat{\sigma}_i, \hat{\Sigma}_{\hat{r}})$
- 6) Compute  $\hat{R}_{\text{opt}} = \sum_{i=1}^{\hat{r}} \omega_{i, \hat{r}}^{\text{opt}} \hat{u}_i \hat{v}_i^T$

Mean  $\mu_x$  of the spectrum  $\tilde{X}$  can be used to get improved de-noising.

$$\mu_x, \hat{r} = \frac{1}{n-\hat{r}} \sum_{i=\hat{r}-1}^n \partial_{\sigma_i}(x_n) \quad (2)$$

The correctness for the estimation of the rank can be tested by using relative mean squared error.

$$\text{MSE}, \hat{r} = \sum_{i=1}^{\hat{r}} \frac{1}{\hat{D}(\hat{\sigma}_i, \hat{\Sigma}_{\hat{r}})} - \sum_{i=1}^{\hat{r}} (\hat{w}_i, r^{\text{opt}})^2 \quad (3)$$

$$\text{rel MSE}, \hat{r} = 1 - \frac{\sum_{i=1}^{\hat{r}} (\hat{w}_i, r^{\text{opt}})^2}{\sum_{i=1}^{\hat{r}} \frac{1}{\hat{D}(\hat{\sigma}_i, \hat{\Sigma}_{\hat{r}})}} \quad (4)$$

The good estimate for the rank is achieved when *rel MSE*,  $\hat{r}$  is near to 0 and estimation is poor when a value near to 1. The decision of the effective rank in step 1 is very important. In [21] it is mentioned that the target sub-space is multidimensional sub-space and not 1-D. These sub-spaces depend upon the target position; its size and number of targets need to be scanned [21]. The strategy adopted in [21] is weak and not adaptive and an alternate strategy is necessary. Assuming ( $R = 3$ ) can be sufficient to find optimum rank using knee-bow technique which is shown in Fig. 3. The developed images are shown in Fig. 4.

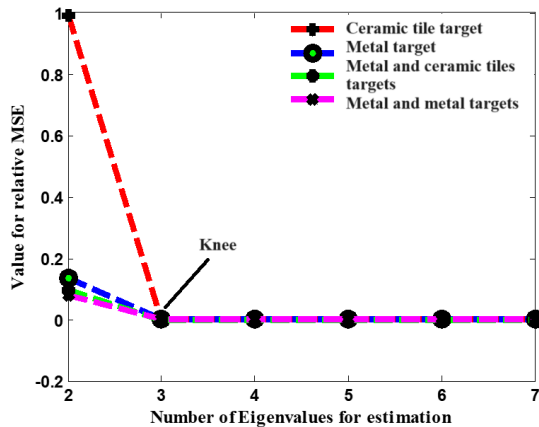


Fig. 3. Plot for the estimation of effective rank.

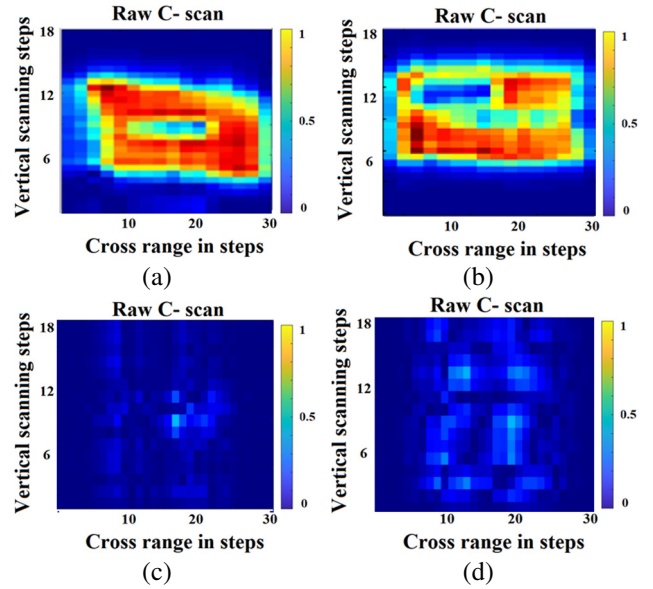


Fig.4. C- scan images using estimated rank  $R = 3$  for targets (a) and (b) for metal targets (c) and (d) for ceramic tile targets

It can be observed from Fig. 4 only effective rank estimation also not sufficient especially in weak reflective target and rank optimization with adaptive thresholding is necessary. Hence, in the next section a novel algorithm for image enhancement is discussed.

#### IV. PROPOSED GA FRAMEWORK FOR RANK OPTIMIZATION AND ADAPTIVE THRESHOLDING

In low rank approximation (LRA) it is known that lower eigenvalues represent the signal and higher eigen values represent the noise. Our objective is to find the optimum rank which can maximize the signal-to-noise ratio (SNR). To address this issue Genetic Algorithm (GA) framework can be useful, it is a process of optimization inspired from natural selection and evolution. Interested readers can refer to [22] for more clarification on Genetic algorithm. According to EYM estimator the rank of *opt-shrink* should be optimum i.e., it should represent only signal by maximizing the SNR. This optimization problem can be represented in the following manner in GA framework.

$$[\hat{R}_{\text{opt}}] = \max_{\hat{r} \geq 1} \sum_{i=1}^{\hat{r}} \omega_{i, \hat{r}}^{\text{opt}} \hat{u}_i \hat{v}_i^T \quad (5)$$

$$\text{i.e., } F(\hat{r}) = [f_1(\hat{r})], 1 \leq (\hat{r}) \leq \quad (6)$$

$$\text{length}(\omega_{i, \hat{r}}^{\text{opt}})$$

$$\text{such that } f_1(\hat{r}) < 1 \text{ i.e., minimum } \text{rel MSE}, \hat{r}$$

After implementing the GA framework, the obtained *rel MSE*,  $\hat{r}_i$  values for different estimated rank are shown in Table III. It can be observed that weak reflected target i.e. low dielectric target is more sensitive to the rank estimation, while strong reflective target is not much. It is also obvious because reflections from the high dielectric targets are strong, and noise does not affect

the signal much and satisfactory SNR is achieved at multiple values of rank.

TABLE III  
rel MSE,  $\hat{\rho}$ s FOR DIFFERENT COMBINATIONS OF LOW AND HIGH DIELECTRIC TARGETS

Estimated rank for Eigen-values.	rel MSE, $\hat{\rho}$ for ceramic tiles target	rel MSE, $\hat{\rho}$ for metal target
2	0.9914	0.1356
3	1.2213e-015	2.4425e-015
4	1.1102e-015	1.9984e-015
5	1.2212e-015	1.9985e-015
6	1.2211e-015	1.9985e-015
7	1.2214e-015	1.6653e-015

Finally, adaptive thresholding implemented using statistics of the image as follows: Coverage probability for the pixel in statistics defined by  $3\sigma$  rule or empirical rule [23]. It states that the pixel values selected from the Gaussian distribution falls within the  $3\sigma$  range i.e.,

$$pr(\mu - 1\sigma) \leq x \leq pr(\mu + 1\sigma) \approx 68.27\% \quad (5)$$

$$pr(\mu - 2\sigma) \leq x \leq pr(\mu + 2\sigma) \approx 95.45\%$$

$$pr(\mu - 3\sigma) \leq x \leq pr(\mu + 3\sigma) \approx 99.73\%$$

Further, Chebyshev inequality [23] defines weaker rule for non – normally distributed pixel values. It's saying that 88.8% of values lie within  $3\sigma$ .

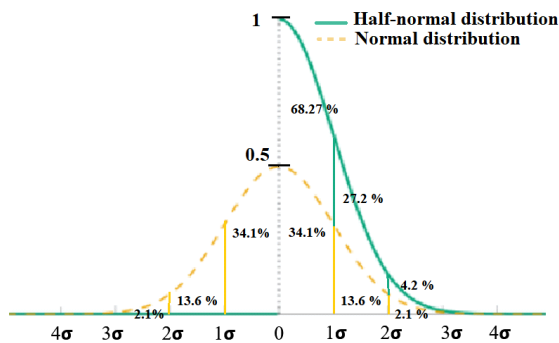


Fig.5. The probability distribution for normal and half-normal distribution [27]

Applying this weaker rule to target images in Fig. 4, the image probability distribution considered as threshold in terms of intensity is given by,

$$th(I_m) = \mu(I_m) + \widetilde{R}_{opt} * \sigma(I_m) \quad (6)$$

Where  $\widetilde{R}_{opt}$  for minimum rel MSE,  $\hat{\rho}$  from Table III,  $\mu$  is the mean,  $\sigma$  is the standard deviation for the  $I_m$  image. The final

images obtained by using this thresholding  $th(I_m)$  and Frangi line enhancement filtering [24] are shown in Fig. 6.

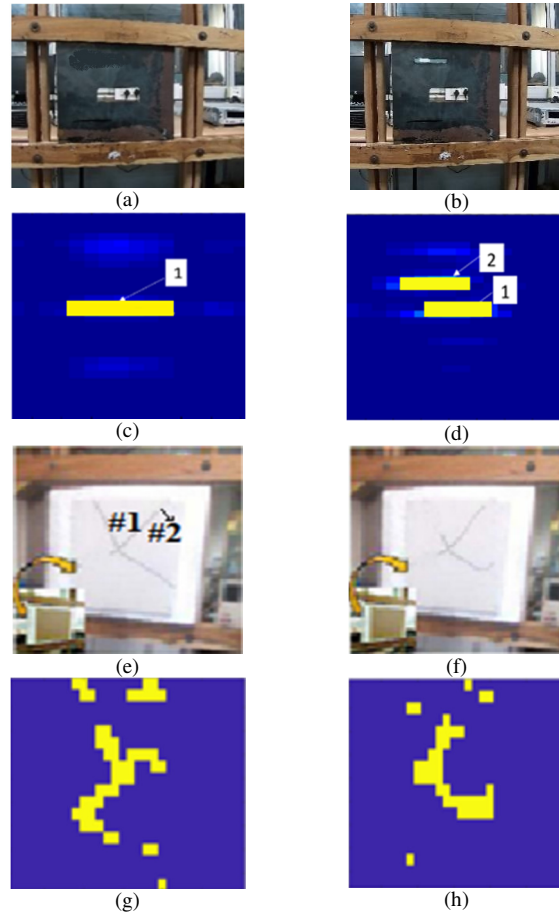


Fig. 6 Final results: (a) metal target with single groove, (b) metal target with double groove, (c) processed image for metal target with single groove (d) processed image for metal target with double groove, (e) ceramic tile with random crack, (f) ceramic tile with diagonal crack, (g) processed image for random crack ceramic tile, (h) processed image for diagonal crack ceramic tile.

## V. CONCLUSION

In this paper we investigate the application of *opt-shrink* algorithm for MMW imaging by optimizing the effective rank. The combinational optimization framework GA optimization is implemented and obtained rank is used for scaling the number of standard deviations for the image. It is observed in our experimental work that low dielectric material is more sensitive for rank selection. The proposed adaptive thresholding is very effective for crack detection in low as well as high dielectric targets. The proposed method can be generalized for all types of materials used in various industries.

## ACKNOWLEDGEMENT

Author likes to express sincere gratitude towards Prof. Dharmendra Singh (IIT Roorkee), Prof. Vivek Sunnapwar (Principal, KJSIT) and Prof. R. K. Shevgaonkar (Provost, Somaiya trust) for constant motivation and providing intellectual support while writing this manuscript.

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