$\alpha\beta$ and sβ-closed functions in Topology *

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Abstract

Aim of this paper is to study the properties of sβ - regular spaces and sβ - closed functions, using β-open (semipreopen sets) sets and semiclosed sets.

1 Introduction

In the literature semiopen sets plays an vital in general topology, due to Normal Levine [11]. In 1965, O. Njastad [21] introduced the concept of α - sets and properties of α - open sets was introduced by A.S. Mashhour et al. [15] the paper contains the notions of α - continuous functions, α - open functions and α - closed functions. In 1983 , the concepts of β - open sets, β - closed sets , β - continuous functions , β - open functions and β - closed functions were introduced and studied by M.E.Abd El-Monsef et al. [1] . D.Andrijevic [4] introduced semipreopen sets and semipreclosed sets respectively and obtained their properties. Later in 1995 Park et al [23] introduced sp-regular spaces. In this paper we define and study the properties of $\alpha\beta$ - closed functions , sβ - closed functions, sβ - regular spaces, composition mappings and results on space preservation.

2 Preliminaries

Throughout the paper (X, τ), (Y, σ) and (Z, γ) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of X, then Cl(A) and Int(A) denote the closure and the interior of A respectively.

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DEFINITION 1 *A subset A of X is said to be*

(i) semiopen*[11] if* $A \subset ClInt(A)$

(ii) α-open*[21] if* $A \subset IntCl(Int(A))$

(iii) semipreopen*[4] if* $A \subset Cl(IntCl(A))$

(iv) gβ - closed (= gsp-closed) *[9]*

if $\beta Cl(A) \subset U$ *whenever* $A \subset U$ *and* U *is open in* X

The complement of a semiopen (resp. α-open, semipreopen (α-open)) set of X is called a semiclosed (resp. α-closed, semipreclosed (α-closed) set in X and the complement of a gβ-closed set is called gβ-open set in X. The family of all semiopen (resp. α-open, β-open) sets of X is denoted by SO(X)(resp. αO(X) , βO(X)) and that of semiclosed (resp. α-closed, β-closed) sets of X is denoted by SF(X) (resp. αF(X)), βF(X)).

DEFINITION 2 *[5, 4, 2] Let A be a subset of a space X, then*

(i) *The intersection of all semi-closed sets containing A is called semi-closure of A and is denoted by sCl (A)*

(ii) *The intersection of all semipreclosed sets containing A is called semipre-closure of A or* β*-closure of A and is denoted by spCl (A) or* $\beta Cl(A)$

DEFINITION 3 *[5, 4, 2] Let A be a subset of a space X, then*

(i) *The semi - interior of A is defined by the union of all semieopen sets contained in A and is denoted by sInt(A).*

(ii) *The semipre - interior of A is defined by the union of all semipreopen sets contained in A and is denoted by spInt(A)*

DEFINITION 4 *A function* $f : X \to Y$ *is called*

(i) *semicontinuous [11] if* $f^{-1}(V)$ *is semiopen in* X *for every open set* V *of* Y

(ii) *semiprecontinuous [16] if* $f^{-1}(V)$ *is semiopen in* X *for every open set* V *of* Y

(iii) *irresolute [6] if* $f^{-1}(V)$ *is semiopen in* X *for every semiopen set* V *of* Y*.*

(iv) β*-irresolute [13] if* $f^{-1}(V)$ *is* β*-open in* X *for every* β*-open set* V *of* Y*.*

(v) α*-irresolute [12] if* $f^{-1}(V)$ *is* α*-open in* X *for every* α*-open set* V *of* Y*.*

(vi) β*-continuous [1] if* $f^{-1}(V)$ *is* β*-open in* X *for every open set* V *of* Y

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DEFINITION 5 *A function $f : X \to Y$ is called*

(i) *semiclosed [22] if $f(F)$ is semiclosed set in Y for each closed set F of X.*

(ii) *presemiclosed [10] if $f(F)$ is semiclosed set in Y for each semiclosed set F of X.*

(iii) *pre - α-open [7] if $f(U)$ is α- open set in Y for each α-open set U of X.*

(iv) *pre - α-closed [7] if $f(U)$ is α- closed set in Y for each α-closed set U of X.*

(v) *pre - β-open [13] if $f(U)$ is β- open set in Y for each β-open set U of X.*

(vi) *pre - β-closed [13] if $f(U)$ is β- closed set in Y for each β-closed set U of X.*

(vii) *presemiopen [6] if $f(U)$ is semiopen set in Y for each semiopen set U of X.*

(viii) *β-closed [1] if $f(F)$ is β-closed set in Y for each closed set F of X.*

(ix) *α-closed [15] if $f(F)$ is α-closed set in Y for each closed set F of X.*

(x) *$\alpha g\beta$ - closed [20] if $f(F)$ is $g\beta$ - closed set in Y for each α-closed set F of X.*

DEFINITION 6 *A topological space (X, τ) is said to be*

(i) *β - regular [3] provided that every closed set F and a point x not in F, can be separated by disjoint β - open sets.*

(ii) *semiregular [8] provided that every semiclosed set F and a point x not in F , can be separated by disjoint semiopen sets.*

(iii) *$\alpha\beta$ -regular [18] provided that every α - closed set F and a point x not in F , can be separated by disjoint β-open sets.*

(iv) *β - normal [13] provided that every pair of non-empty disjoint closed sets can be separated by disjoint β - open sets.*

(v) *$\alpha\beta$ - normal [18] provided that every pair of non-empty disjoint α-closed sets can be separated by disjoint β - open sets.*

(vi) *α T_1-space [14] if for any distinct pair of points x and y in X, there is an α-open set U in X containing x but not y and an open set V in X containing y but not x resp., with $U \cap V \neq \emptyset$.*

3 $\alpha\beta$ - closed functions

DEFINITION 7 *A function $f : X \to Y$ is said to be $\alpha\beta$-closed if the image of each α-closed set of X is β-closed in Y.*

EXAMPLE 3.1 Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $Y = \{1, 2, 3, 4, 5\}$, $\sigma = \{\emptyset, \{1\}, \{2, 3\}, \{1, 2, 3\}, Y\}$ Define $f : X \to Y$ by $f(\mathrm{a}) = 1$, $f(\mathrm{b}) = 2$, $f(\mathrm{c}) = 3$, $f(\mathrm{d}) = f(\mathrm{e}) = 5$ clearly, f is $\alpha\beta$-closed function.

THEOREM 3.2 *A function $f : X \to Y$ is $\alpha\beta$-closed if and only if any subset A of Y and any α-open set F containing $f^{-1}(A)$, there is a β-open set G of Y containing A such that $f^{-1}(G) \subset F$.*

Proof. Necessity. Suppose $f : X \to Y$ is $\alpha\beta$-closed function and $A \subset Y$ and F is α-open set of X containing $f^{-1}(A)$. Put $G = Y - f(X - F)$. Then G is β-open set of Y, $A \subset G$ and $f^{-1}(G) \subset F$

Sufficiency. Let K be any α-closed set of X. Put $B = Y - f(K)$ then $f^{-1}(A) \subset X \setminus K$ and $X - K$ is α-open in X. There exists a β-open set G of Y such that $A = Y - f(K) \subset G$ and $f^{-1}(G) \subset X - K$

Therefore $f(K) = Y - G$ and hence $f(K)$ is β-closed in Y

Some results on $\alpha\beta$ - closed functions and allied mappings :

THEOREM 3.3 *If $f : X \to Y$ and $g : Y \to Z$ be two functions such that $gof : X \to Z$ is $\alpha\beta$-closed function:*

(i) *If f is α-irresolute surjection then g is $\alpha\beta$-closed*

(ii) *If g is β-irresolute injection then f is $\alpha\beta$-closed*

Proof. (i) Let U be an arbitrary α-closed set in Y. Since $g \circ f$ is $\alpha\beta$-closed and f is α-irresolute and surjective then $g(U) = g \circ f(f^{-1}(U))$ is a β - closed set in Z. Hence, g is $\alpha\beta$-closed.

(ii) Since g is injective, we remark that $f(A) = g^{-1}(g(f(A))$ for every subset A of X. Let U be an arbitrary α-closed set in X, then by hypothesis $(g \circ f)(U)$ is a β - closed set in Z. Again $f(U) = g^{-1}(g \circ f)(U))$ is β - closed in Y. Since g is β -irresolute and injective. This shows $f(U)$ is β - closed in Y. Hence, f is $\alpha\beta$ - closed

Theorem 3.4 *: If $f : X \to Y$ is pre - α-closed and $g : Y \to Z$ is $\alpha\beta$ closed then $gof : X \to Z$ is $\alpha\beta$ -closed*

Proof. Let U be an arbitrary α -closed set in X. Since f is pre-α -closed, $f(U)$ is α -closed in Y. Again, since g is $\alpha\beta$ closed and $f(U)$ is α -closed, $g(f(U)) = (g \circ f)(U)$ is β-closed in Z. This shows $g \circ f$ is $\alpha\beta$ -closed.

Definition 8 *A function $f : X \to Y$ is said to be quasi-α - closed function if the image of each α-closed set of X is closed in Y.*

Example 3.5 *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \ X\}$ and $Y = \{1, 2, 3\}$, $\sigma = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \ Y\}$. Define $f : X \to Y$ by $f(a) = 1$, $f(b) = f(c) = 2$, $f(d) = f(e) = 3$. Then f is quasi-α-closed function.*

Theorem 3.6 *: If $f : X \to Y$ is quasi - α - closed and $g : Y \to Z$ is β - closed then $gof : X \to Z$ is $\alpha\beta$ - closed*

Proof. straight forward.

4 sβ - closed functions

Definition 9 *A function $f : X \to Y$ is said to be sβ - closed function if the image of each semiclosed set of X is β-closed in Y.*

Example 4.1 *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \ X\}$ and $Y = \{1, 2, 3, 4, 5\}$, $\sigma = \{\emptyset, \{1\}, \{2, 3\}, \{1, 2, 3\}, Y\}$ Define $f : X \to Y$ by $f(a) = 1$, $f(b) = f(c) = 2$, $f(d) = f(e) = 4$. clearly, f is sβ-closed function.*

THEOREM 4.2 *A function $f : X \to Y$ is $s\beta$-closed if and only if any subset A of Y and any α-open set F of X containing $f^{-1}(A)$, there is a β-open set G of Y containing A such that $f^{-1}(G) \subset F$.*

Proof. Is similar to Theorem 3.2

THEOREM 4.3 *If $f : X \to Y$ and $g : Y \to Z$ be two functions such that $gof : X \to Z$ is $s\beta$-closed function:*

(i) *If f is irresolute surjection then g is $s\beta$-closed*

(ii) *If g is β-irresolute injection then f is $s\beta$-closed*

Proof. Is similar to Theorem 3.3

THEOREM 4.4 *: If $f : X \to Y$ is presemiclosed and $g : Y \to Z$ is $s\beta$ -closed then $gof : X \to Z$ is $s\beta$ -closed*

Proof. straight forward.

DEFINITION 10 *A function $f : X \to Y$ is said to be quasi-semiclosed function if the image of each semiclosed set of X is closed in Y.*

EXAMPLE 4.5 *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $Y = \{1, 2, 3\}$, $\sigma = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, Y\}$. Define $f : X \to Y$ by $f(a) = 1$, $f(b) = f(c) = 2$, $f(d) = f(e) = 4$. Then f is quasi-semiclosed function.*

THEOREM 4.6 *: If $f : X \to Y$ is quasi - semiclosed and $g : Y \to Z$ is β - closed then $gof : X \to Z$ is $s\beta$ - closed*

Proof. straight forward.

REMARK 4.7 *: The functions defined above have following implications :*

$$\begin{tikzcd}
\text{pre-}\beta\text{ - closed} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \beta\text{ - closed} \\
s\beta\text{ - closed} \arrow[r, Rightarrow] & \alpha\beta\text{ - closed} \arrow[u, Rightarrow] \\
\text{quasi - semiclosed} \arrow[r, Rightarrow] \arrow[u, Rightarrow] \arrow[d, Rightarrow] & \text{quasi - }\alpha\text{ - closed} \arrow[u, Rightarrow] \arrow[d, Rightarrow] \\
\text{presemiclosed} \arrow[d, Rightarrow] \arrow[uu, bend left] & \text{pre - }\alpha\text{ - closed} \arrow[l, Rightarrow] \arrow[d, Rightarrow] \arrow[uu, bend right] \\
\text{semiclosed} & \alpha\text{ - closed} \arrow[l, Rightarrow]
\end{tikzcd}$$

None of the implications in the above are reversible.

EXAMPLE 4.8 *Let* $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{a, b\}, \ X\}$ *and* $Y = \{a, b, c, d\}$, $\sigma = \{\emptyset, \{a, b\}, \{a, b, c\}, \ Y\}$. *Define* $f : X \to Y$ *by* $f(a) = d$, $f(b) = b$, $f(c) = c$, *Then* f *is pre-*α*-closed but not quasi-semiclosed.*

EXAMPLE 4.9 *Let* $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{a, b\}, \ X\}$ $Y = \{a, b, c, d\}$, $\sigma = \{\emptyset, \{a, b\}, \{a, b, c\}, \ Y\}$. *Define* $f : X \to Y$ *by* $f(a) = d$, $f(b) = b$, $f(c) = d$, *Then* f *is quasi-semiclosed and pre-*β*-closed*

5 sβ - regular spaces

DEFINITION 11 *A topological space* (X, τ) *is said to be* $s\beta$ *- regular for each semiclosed set* F *and a point* x *in* $X - F$, *there exist disjoint* β *- open sets* U *and* V *such that* $x \in U$ *and* $F \subset V$

EXAMPLE 5.1 *Let* $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, b\}, \{a, b, c\}, \ X\}$, *the space* (X, τ) *is* $s\beta$*-regular but not semiregular.*

REMARK 5.2 *: The following implications hold ingeneral :*

$$\begin{array}{ccc} s\beta \text{ - regular space} & \Longrightarrow & \alpha\beta \text{ - regular} \\ \Uparrow & & \Downarrow \\ \text{semiregular space} & & \beta \text{ - regular space} \end{array}$$

None of the implications in the above are reversible.

THEOREM 5.3 *For the topological space* (X *the following statements are equivalent :*

(i) *X is sβ-regular*

(ii) *For each $x \in X$ and for each semiopen set U containing x there exists a β-open set V containing x such that $x \in V \subset \beta Cl(V) \subset U$*

(iii) *For each semiclosed set F of X, $\cap \{\beta\ Cl(V) \ / \ F \subset V \in \beta O(X)\} = F$*

(iv) *For each nonempty subset A of X and each $U \in SO(X)$ if $A \cap U \mp \emptyset$ then there exists $V \in \beta\ O(X)$ such that $A \cap V \mp \emptyset$ and $\beta Cl(V) \subset U$*

(v) *For each nonempty subset A of X and each $F \in SF(X)$ if $A \cap F = \emptyset$ then there exists disjoint β - open sets V, W such that $A \cap V \mp \emptyset$ and $F \subset V$*

Proof. (i)$\rightarrow$ (ii) : Let X is sβ- regular space. Let x$\in$ X and U be semiopen set containing x implies $X - U$ is semiclosed such that x $\notin X - U$. Therefore by (i) there exists β open sets V and W such that x$\in V$ and $X - U \subset W$ whcih implies $X - W \subset U$. Since V and W disjoint $\beta Cl(V) \cap W = \emptyset$ results $\beta Cl(V) \subset X - W \subset U$. Therefore $x \in$V$\subset \beta Cl(V) \subset U$.

(ii) $\rightarrow$ (iii) : Let F be a semiclosed subset of X and $x \notin F$. then $X - F$ is semiopen set containing x. By (ii) there exists β-open set U such that $x \in U \subset \beta Cl(U) \subset X - F$ implies $F \subset X - \beta Cl(U) \subset X - U$. which means $F \subset V \subset X - U$ where $V = X - \beta Cl(U) \in \beta O(X)$ and $x \notin V$ that implies $x \notin \beta Cl(V)$ $x \notin \cap \{\beta\ \mathrm{Cl}(V) \ / \ F \subset V \in \beta O(X)\}$. Therefore
$\cap \{\beta\ \mathrm{Cl}(V) \ / \ F \subset V \in \beta O(X)\} = F$

(iii) $\rightarrow$ (iv) : A be a subset of X and $U \in SO(X)$ such that $A \cap U \neq \emptyset$ there exist $x_0 \in X$ such that $x_0 \in A \cap U$ Therefore $X - U$ is semiclosed set not containing x_0 implies $x_0 \notin sCl(X - U)$. By (iii) there exists $W \in \beta O(X)$ such that $X - U \subset W$ results $x_0 \notin \beta Cl(W)$.

Put $V = X - \beta Cl(W)$, then V is β open set containing x_0 which implies $A \cap V \neq \emptyset$ and $\beta Cl(V) \subset \beta Cl(X - \beta Cl(W)) \subset \beta Cl(X - W)$. Therefore $\beta Cl(V) \subset \beta Cl(X - W) \subset U$.

(iv) $\rightarrow$ (v) : Let A be a nonempty subset of X and F be semiclosed set such that $A \cap F = \emptyset$. Then $X - F$ is semiopen in X and $A \cap (X - F) \neq \emptyset$ Therefore by (iv), there exist $V \in \beta O(X)$ such that $A \cap V \neq \emptyset$ and $\beta Cl(V) \subset X - F$. Put $W = X - \beta Cl(V)$ then $W \in \beta O(X))$ such that $F \subset W$ and $W \cap V = \emptyset$.

(v) $\rightarrow$ (i) : Let $x \in X$ be arbitrary and F be semiclosed set not containing x. Let $A = X - F$ be a nonempty semiopen set containing x then by (v) there exist disjoint β open sets V and W such that $F \subset W$ and $A \cap V \neq set$ implies $x \in V$. Thus X is β - regular.

Some space preservation results in the following :

THEOREM 5.4 *If If $f : X \rightarrow Y$ is a pre - β-open , irresolute bijection and X is sβ-regular space , then Y is sβ-regular.*

Proof. : Let F be any semiclosed subset of Y and $y \in Y$ with $y \notin F$. Since f is irresolute, $f^{-1}(F)$ is semiclosed set in X. Again, f is bijective, let $f(x) = y$, then $x \notin f^{-1}(F)$. Since X is sβ- regular , there exist disjoint β-open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$. Since f is , pre - β-open bijection, we have $y \in f(U)$ and $F \subset f(V)$ and $f(U) \cap f(V) = f(U \cap V) = \emptyset$ Hence, Y is sβ-regular.

THEOREM 5.5 *If If $f : X \rightarrow Y$ is pre -semiclosed , β-irresolute injection and Y is sβ-regular space, then X is sβ-regular .*

Proof. : Let F be any semiclosed set of X and $x \notin F$. Since f is presemiclosed injection, $f(F)$ is semiclosed set in Y and $f(x) \notin f(F)$. Since Y is sβ-regular space and so there exist disjoint β-open sets U and V in Y such that $f(x) \in U$ and $f(F) \subset V$. By hypothesis, $f^{-1}(U)$ and $f^{-1}(V)$ are β-open sets in X with $x \in f^{-1}(U)$, $F \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence X is s$\beta$-regular. X is s$\beta$-regular

THEOREM 5.6 *Every α- T_1 and $\alpha\beta$-normal space is $\alpha\beta$-regular*

Proof. : Let X be a α- T_1 -space and $\alpha\beta$-normal space. Let F be any α-closed set in X and $x \in X - F$. As X is α- T_1 -space, {x} is α-closed for all $x \in X$. Thus F and $\{x\}$ are two disjoint α-closed sets in X. Since X is $\alpha\beta$-normal space, there exist disjoint β-open sets G and H in X such that $\{x\} \in G$ and $F \subset H$ implies $x \in G$ and $F \subset H$. This shows X is $\alpha\beta$-regular.

THEOREM 5.7 *If $f : X \to Y$ is a α-irresolute, $\alpha g\beta$-closed surjection and X is $\alpha\beta$ - normal then so is Y.*

Proof. : Let A and B be any two disjoint α-closed sets in Y.Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint α-closed sets of X. Since f is α-irresolute , $\alpha\beta$ - normal, there exist β - open sets U and V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. By theorem [18], there exist gβ - open sets G and H of Y such that $A \subset G$ and $B \subset H$, $f^{-1}(G) \subset U$ and $f^{-1}(H) \subset V$. Then $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. Thus $G \cap H = \emptyset$. By theorem [18] Y is $\alpha\beta$ - normal.

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