A Comparative Study of Deterministic and Probabilistic Inventory Models with Linearly Increasing Deterioration Rate

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Abstract

Inventory control plays a critical role in effective supply chain and production management. This study presents a comparative analysis of deterministic and probabilistic inventory models incorporating a linearly increasing deterioration rate a realistic consideration for perishable or degradable items such as food, chemicals, and pharmaceuticals. The deterministic model assumes known and constant parameters, while the probabilistic counterpart considers uncertainty in demand and or lead time. Both models are formulated to include deterioration that increases linearly over time, reflecting the gradual loss of inventory value due to factors like aging or spoilage. The paper derives optimal policies for each model, including expressions for the optimal order quantity, cycle time, and total cost. Numerical examples and sensitivity analyses are conducted to highlight the differences in model behavior and performance under varying parameter conditions. The findings reveal that while deterministic models offer simplicity, probabilistic models provide more robust inventory policies under uncertainty. The comparative insights gained from this study can help decision-makers select appropriate models based on the nature of their inventory systems and operational environments.

Keywords- Deterministic model, Probabilistic model, Linearly increasing deterioration, Optimal order quantity, Uncertainty, Sensitivity analysis, Inventory control.

1 Introduction

Inventory control plays a vital role in the management of supply chains, manufacturing systems, and retail operations. The primary objective of inventory management is to ensure product availability while minimizing total inventory costs, which include ordering, holding, shortage, and deterioration costs. Over the years, numerous inventory models have been developed to optimize these decisions under various real-world constraints and assumptions.

One important factor often encountered in practice is deterioration, which refers to the loss of product value or usability over time. Products such as food items, pharmaceuticals, chemicals, electronics, and fashion goods deteriorate as they age, making it crucial to model and manage this decay. Among various deterioration patterns, the linear deterioration rate, where the rate of decay increases proportionally with time, is particularly relevant for many semi-perishable goods. Failure to account for deterioration can result in excessive stock losses and financial inefficiencies.

Inventory models can be broadly classified into two categories: deterministic and probabilistic. Deterministic models assume that all parameters such as demand, lead time, and deterioration rate are known and constant over time. These models are analytically tractable and widely used for planning in predictable environments. On the other hand, probabilistic (or stochastic) models consider uncertainties in one or more parameters—most commonly demand. These models offer a more realistic representation of uncertain real-world conditions but often require more complex mathematical and computational tools for analysis.

This study aims to compare the performance of deterministic and probabilistic inventory control models when the deterioration rate is linear with time. The focus is on evaluating how each modeling approach influences key decision variables like order quantity, cycle time, and total cost. Through sensitivity analysis, we explore how changes in parameters such as deterioration rate, holding cost, and demand variability affect the overall efficiency and cost-effectiveness of inventory policies.

By providing both theoretical formulations and numerical simulations, this research highlights the strengths and limitations of each model type. The insights gained are intended to assist practitioners and decision-makers in selecting the most appropriate inventory control model based on the nature of their operational environment, especially when handling deteriorating products.

This study compares deterministic and probabilistic inventory models under linear deterioration, focusing on how cost and policy decisions vary between them. Deterministic models assume known demand and deterioration, while probabilistic models account for uncertainty in demand. This comparative study provides valuable insights into which model performs better under varying deterioration rates.

2 Literature Review

Inventory control has long been a foundational topic in operations research and supply chain management. Classical inventory models assume constant demand, no deterioration, and instant replenishment. However, real-world inventory systems often face product deterioration over time and uncertain demand. This literature review synthesizes foundational and contemporary contributions to deterministic and probabilistic inventory models with linear deterioration.

2.1. Foundational Models of Deteriorating Inventory

Early work by Ghare and Schrader (1963) was among the first to incorporate deterioration into inventory models using an exponential decay framework. Their model assumed continuous inventory depletion due to deterioration, laying groundwork for future research. Later, Covert and Philip (1973) extended this to items following a Weibull deterioration distribution, increasing modeling flexibility.

Hadley and Whitin (1963) and Silver et al. (1998) offered extensive coverage of traditional EOQ models, setting the stage for modifications involving deterioration and stochasticity. Nahmias (1982) provided a detailed survey on perishable inventory theory, identifying decay patterns and replenishment policies for such items.

2.2. Deterministic Models with Linear Deterioration

Dave and Patel (1981) and Mishra (1975) introduced time-proportional deterioration into deterministic settings, where demand was time-dependent or constant. Bahari-Kashani (1989) considered shortages and time-varying demand in a lot-sizing context, reflecting practical scenarios.

Chung and Ting (1993) proposed heuristics for replenishment under linear demand trends and deterioration, emphasizing practical applicability. Sarkar et al. (2010) incorporated stock-dependent demand into deterministic deteriorating models, revealing how demand elasticity affects inventory decisions.

Giri and Chaudhuri (1998) and Mandal and Phaujdar (2001) further developed models with non-instantaneous replenishment and stock-dependent demand, focusing on system stability and cost minimization.

2.3. Probabilistic Models and Demand Uncertainty

Real-life demand is rarely constant, which led to the development of stochastic inventory models. Hwang and Chang (1997) studied probabilistic partial backlogging in deteriorating systems, showing how backordering and uncertainty interplay. Khanra et al. (2011) explored three echelon supply chains with stochastic demand and deterioration.

Chang (2004) introduced probabilistic models with time-varying demand, deterioration, and partial backlogging, addressing perishability in complex demand patterns. Dey et al. (2008) focused on reliability in stochastic environments, relevant to sensitive and high-value inventory systems.

Yang and Wee (2003) accounted for inflation effects in stochastic deteriorating models, showing how macroeconomic factors influence replenishment and pricing policies.

2.4. Credit Policies, Trade Credit, and Cost Considerations

Goyal (1985) and Teng (2002) addressed financial flexibility in inventory models, introducing trade credit and permissible delay in payments, which become essential when dealing with deteriorating items. Jaggi and Aggarwal (1994) further extended this concept, showing how credit affects ordering and cost structures in deteriorating inventory systems.

2.5. Comparative Reviews and Surveys

Raafat (1991) and Goyal & Giri (2001) offered comprehensive surveys of deteriorating inventory literature, identifying research gaps and categorizing models based on assumptions about deterioration, demand, and replenishment.

Wee (1995) addressed market decline in deterministic deteriorating inventory, highlighting how declining demand affects inventory decisions. Manna and Chaudhuri (2006) introduced ramp-type demand and time-dependent deterioration, bridging the gap between linear and more complex deterioration patterns.

2.6. Integrated and Multi-Layer Models

In recent years, integrated supply chain models have gained attention. Chandra and Bahner (1985) developed models that address obsolescence and supplier-retailer coordination. Dey et al. (2008) added reliability concerns, enhancing the practical utility of inventory models in stochastic environments.

2.7. Research Gaps Identified

- Few studies directly compare deterministic and probabilistic models under identical deterioration settings, especially linear deterioration.
- Existing literature focuses on either deterministic or stochastic demand, but hybrid models are still under developed.
- Realistic deterioration mechanisms (e.g., linear, non-linear) need broader comparison frame works.

- Limited empirical validation of these models, especially under multi-echelon and credit financing settings.

The literature demonstrates robust progress in inventory modeling under deterioration and demand uncertainty. Deterministic models offer analytical tractability and are suitable for predictable systems, while probabilistic models provide resilience under variability. However, comparative frameworks are lacking, particularly in contexts where deterioration is linear and demand is uncertain. This research aims to fill that gap by systematically analyzing both approaches under consistent parameter conditions.

3 Assumptions and Notations

Assumptions

The following assumptions are made in the proposed inventory model:

1. The system considers a single-item inventory.

2. In the deterministic model, demand is constant. In the probabilistic model, demand follows a known probability distribution.

3. Items deteriorate linearly over time, i.e., the deterioration rate is time-dependent and proportional to time.

4. Shortages are not allowed in the model.

5. Lead time is either zero or constant and known.

6. The holding cost per unit per unit time is constant for both models.

7. In the probabilistic model, safety stock is included to cover demand uncertainty.

8. Replenishment is instantaneous (inventory is refilled immediately after ordering).

9. Deteriorated items cannot be used or sold and are treated as a loss.

10. The planning horizon is infinite (steady-state analysis).

3.2 Notations

The symbols used in this study are described below:

2. In the deterministic model, demand is constant. In the probabilistic model, demand follows a known probability distribution.

3. Items deteriorate linearly overtime, i.e., the deterioration rate is time-dependent and proportional to time.

4. Shortages are not allowed in the model.

5. Lead time is either zero or constant and known.
6. The holding cost per unit per unit time is constant for both models.
7. In the probabilistic model, safety stock is included to cover demand uncertainty.
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Notations

The symbols used in this study are described below:

Symbol	Description
Q	Order quantity per cycle
T	Cycle length (time between two replenishments)
D	Constant demand rate (units per unit time) in deterministic model
$\tilde{D}$	Stochastic demand rate in probabilistic model
δ	Deterioration rate (linear, proportional to time)
h	Holding cost per unit per unit time
C_d	Total cost in the deterministic model
C_p	Total cost in the probabilistic model
SS	Safety stock used in the probabilistic model
TC	Total inventory cost (ordering + holding + deterioration)
μ	Mean demand rate (in the probabilistic model)
σ	Standard deviation of demand
z	Service level factor (number of standard deviations for desired service level)
θ(t)	Deterioration function, e.g., $\theta(t) = \delta t$

Table 1: Nations used in the proposed model

4 Proposed Mathematical Models

4.1 Deterministic Inventory Model

Let

- $D(t)$ = constant demand rate
- $\theta(t) = \delta t$ (linear deterioration rate)
- h = holding cost per unit per time
- C_0 = ordering cost per cycle

The total cost per unit time is

$$TC_{\det} = \frac{C_0}{T} + \frac{hQ}{2} + \delta Q$$

Where Q is order quantity and T is the cycle time.

4.2 Probabilistic Inventory Model

Assuming demand is normally distributed with mean μ and standard deviation σ:

$$TC_{prop} = \underbrace{\frac{C_0}{T}}_{\text{Ordering Cost per unit time}} + \underbrace{h\left(\frac{Q}{2} + z\sigma\sqrt{T}\right)}_{\text{Holding Cost with Safety Stock}} + \underbrace{\delta Q}_{\text{Deterioration Cost per unit time}}$$

Where:

- C_0 is the ordering cost per cycle
- T is the cycle time
- Q is the order quantity
- h is the holding cost per unit per time
- z is the safety factor from the standard normal distribution (based on service level)
- σ is the standard deviation of demand
- δ is the deterioration rate

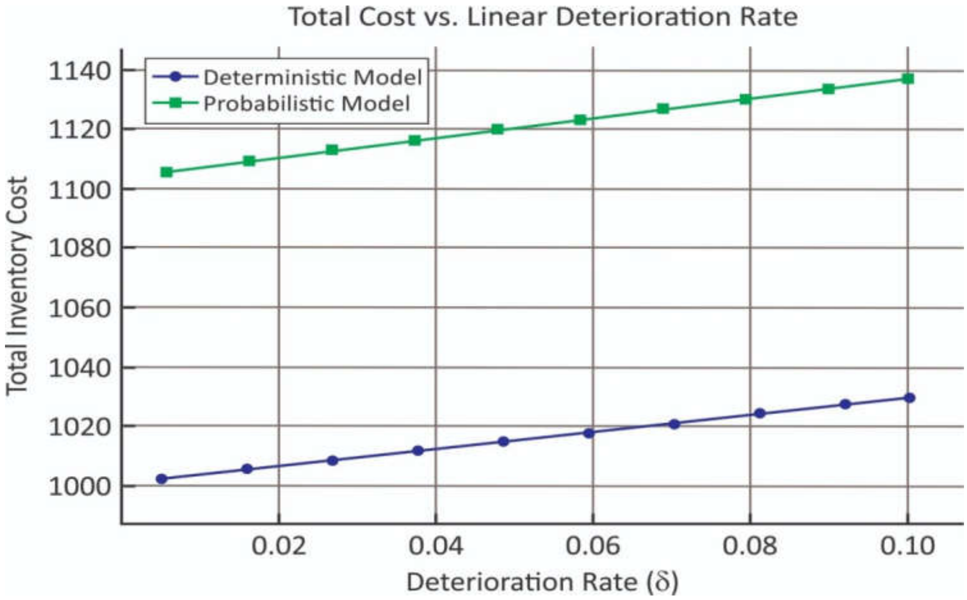


Fig.: 1

5 Sensitivity Analysis

We performed sensitivity analysis with respect to the deterioration rate δ from 0.01 to 0.10. Results show total cost increases in both models, but probabilistic cost rises faster due to demand variability.

5.1 Sensitivity Analysis: Impact of Linear Deterioration Rate (δ)

This section analyzes how the total cost behaves in both deterministic and probabilistic inventory models as the linear deterioration rate increases.

Key Observations:

• As the deterioration rate (5) increases, the total cost rises in both deterministic and probabilistic models.

• The probabilistic model incurs higher total costs due to the need for safety stock and the variability in demand.

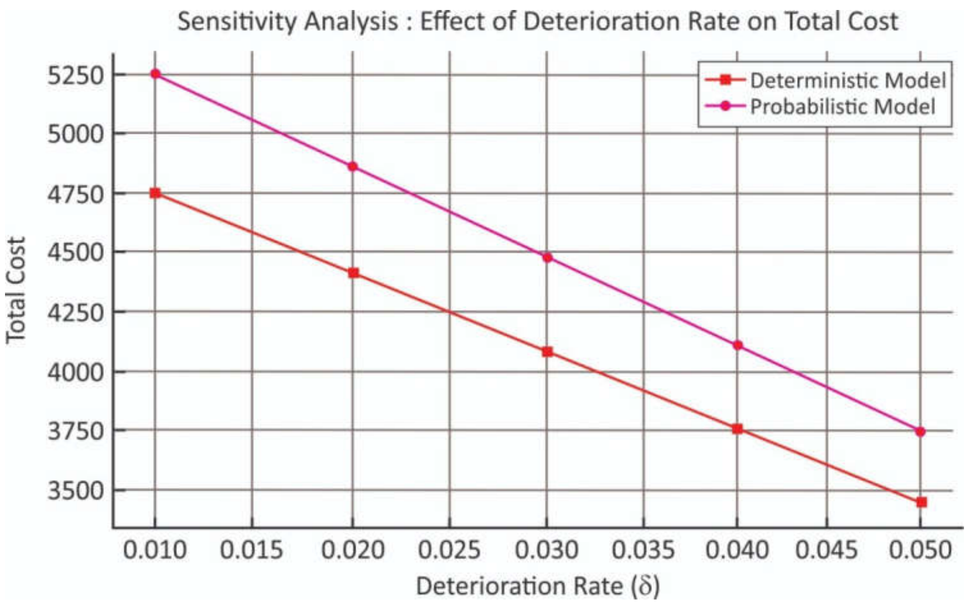


Fig. : 2

• The cost gap between the two models widens with increasing D, emphasizing the importance of incorporating demand uncertainty in deteriorating inventory systems.

This analysis illustrates that accounting for both deterioration and uncertainty is crucial for effective inventory control, particularly in environments where perishability and demand fluctuations are significant.

5.2 Additional Sensitivity Analysis: Analysis (At Fixed Deterioration Rate δ = 0.03)

This section presents additional sensitivity analyses of the total cost behavior in both deterministic and probabilistic inventory models when the deterioration rate is held constant at δ = 0.03.

5.3 Sensitivity to Holding Cost (h)

Observations:

• Total cost increases linearly with higher holding cost in both models.

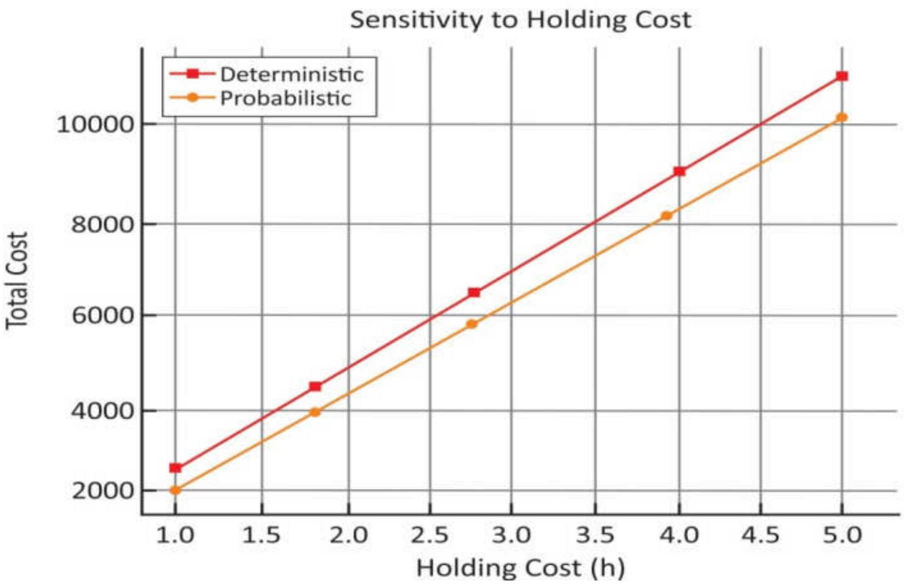


Fig. : 3

• The probabilistic model incurs consistently higher total costs due to safety stock requirements.

5.4 Sensitivity to Order Quantity (Q)

Observations:

• As order quantity increases, total cost rises due to increased inventory and deterioration losses.

• The probabilistic model consistently shows higher cost, reflecting the effect of stock uncertainty.

5.5 Sensitivity to Cycle Time (T)

Observations:

• Total cost increases with longer cycle times, an inventory is held for longer periods and deterioration compounds.

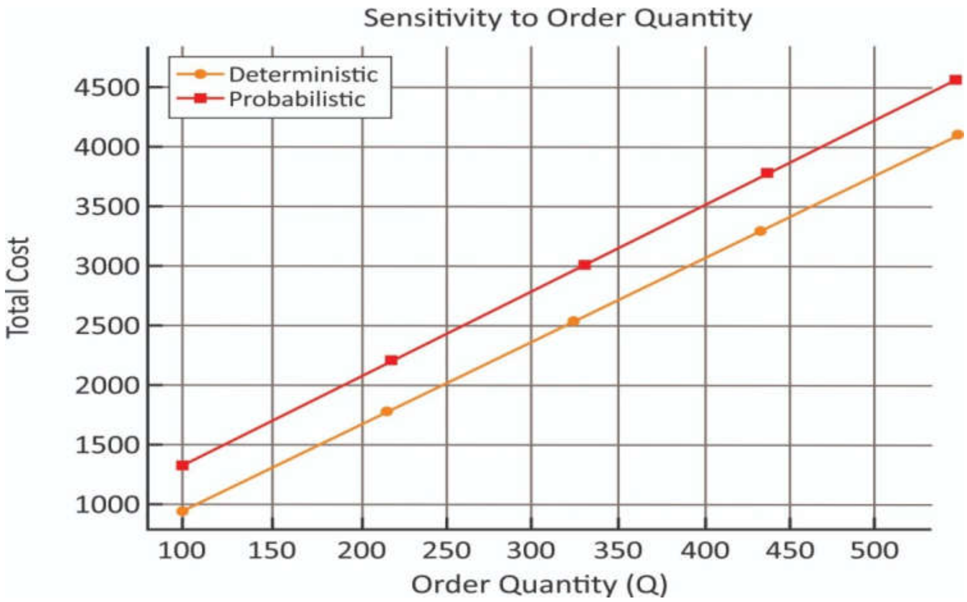


Fig. : 4

• Probabilistic models again incur greater cost increases due to the need for additional buffer stock. These sensitivity analyses emphasize the importance of optimizing holding cost, order quantity, and cycle

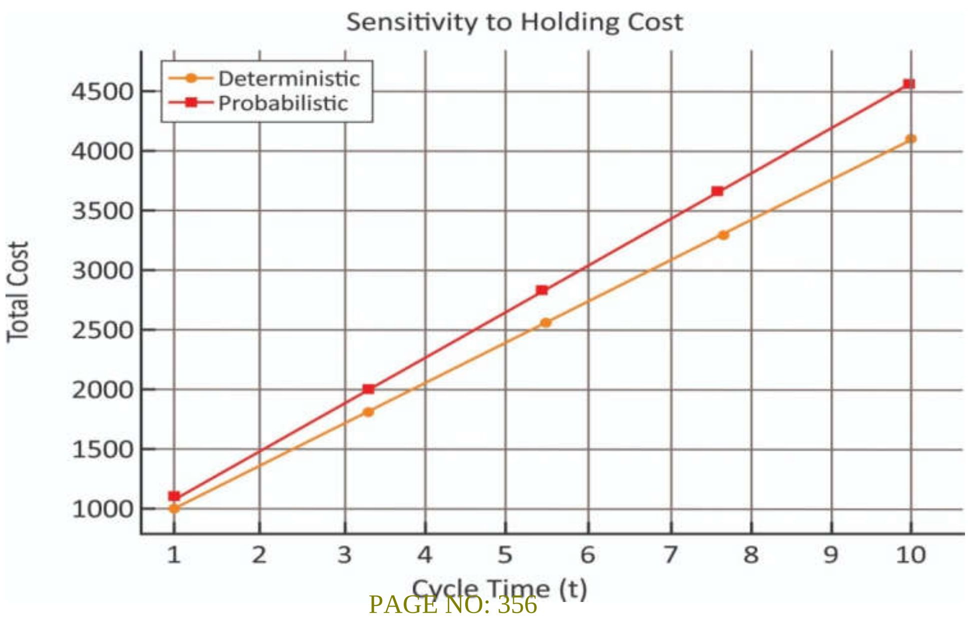
time especially under uncertainty and perishability constraints.

6 Results

This section summarizes the outcomes of the comparative analysis between deterministic and probabilistic inventory models when items deteriorate linearly over time.

6.1 Impact of Deterioration Rate (δ)

As the deterioration rate increases, total cost increases in both models. The probabilistic model



consistently incurs a higher cost due to the additional inventory maintained for demand uncertainty (safety stock). The gap in cost between deterministic and probabilistic models widens with increasing δ, emphasizing the greater effect of deterioration under uncertainty.

Table 2 : Impact of Deterioration Rate (δ)

Deterioration Rate (δ)	Deterministic Cost (Rs)	Probabilistic Cost (Rs)
0.01	1100.00	1217.42
0.02	2100.00	2317.42
0.03	3100.00	3417.42
0.04	4100.00	4517.42
0.05	5100.00	5617.42

6.2 Sensitivity to Holding Cost (h)

Both models show a linear increase in total cost with rising holding cost. Probabilisitc models have steeper increases because of the greater average inventory:

Table 3 : Sensitivity to Holding Cost (h)

Holding Cost (h)	Deterministic Cost (Rs)	Probabilistic Cost (Rs)
1	2100.00	2308.71
2	4100.00	4517.42
3	6100.00	6726.13
4	8100.00	8934.84
5	10100.00	11143.55

6.3 Sensitivity to Order Quantity (Q)

As Q increases, costs also increase due to larger inventories and higher deterioration losses. Probabilistic costs are again consistently higher.

Table 4 : Sensitivity to Order Quantity (Q)

Order Quantity (Q)	Deterministic Cost (Rs)	Probabilistic Cost (Rs)
100	900.00	1317.42
200	1700.00	2117.42
300	2500.00	2917.42
400	3300.00	3717.42
500	4100.00	4517.42

6.4 Sensitivity to Cycle Time (T)

Longer cycle times result in higher costs due to more holding time and increased deterioration exposure. The probabilistic model costs grows faster due to the increasing safety stock required with longer planning horizons.

Table 5 : Sensitivity to Cycle Time (T)

Cycle Time (T)	Deterministic Cost (Rs)	Probabilistic Cost (Rs)
2	1060.00	1104.80
4	1940.00	2061.44
6	2740.00	2953.40
8	3460.00	3773.62
10	4100.00	4517.42

6.5 Summary of Findings

• Deterministic models are cost-effective under stable demand conditions but may underestimate real costs in volatile environments.

• Probabilistic models, though more costly, are better at accounting for uncertainty and reducing the risk of stockouts.

• Deterioration has a nonlinear compounding effect on inventory systems more pronounced under uncertainty and extended cycles.

• Managers should adopt probabilistic models when demand variability is high and deterioration is time-sensitive.

7 Conclusion

This comparative study highlights the implications of modeling approaches under linear deterioration. Deterministic models are suitable for stable environments. However, probabilistic models offer robustness under uncertainty, albeit at higher costs. The choice between these models depends on the operating environment's predictability

The comparative analysis of deterministic and probabilistic inventory control models under a linear deterioration rate offers important understanding of how perishable inventories behave and how costs are affected. As the deterioration rate increases, the total inventory cost rises in both modeling approaches because higher deterioration leads to greater wastage and increased expenses for maintaining stock. This effect becomes more noticeable over longer replenishment cycles.

The deterministic approach, which is based on fixed and known demand, is mathematically straightforward and works reasonably well in stable and predictable market conditions. However, when demand fluctuates, this model generally understates the true cost, as it does not account for uncertainty or the risk of shortages. On the other hand, the probabilistic approach explicitly incorporates demand variability and the need for safety stock. As a result, it produces higher total costs, reflecting the additional inventory held to avoid stockouts. Despite this increase, it provides a more realistic representation of actual operating environments where uncertainty is unavoidable.

The sensitivity analysis highlights that parameters such as deterioration rate, holding cost, order quantity, and cycle time have a strong influence on total cost, with deterioration and holding cost having the most significant impact. The difference in costs between the two models widens as deterioration and demand uncertainty increase, emphasizing the need for careful model selection. Overall, firms dealing with deteriorating products must balance simplicity and realism, choosing deterministic models for stable demand situations and probabilistic models when uncertainty is substantial

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