Bimatrix Game-Theoretic Models of Capital Structure: Nash, Stackelberg, and Germeier Equilibria

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Abstract: *This paper proposes two types of game-theoretic models of capital structure. One is based on the individual rationality of players, while the other assumes a possible hierarchical structure in the relationship between Player I and Player II, who represent the owner and manager of a group of companies within the same industrial segment. The elements of the corresponding payoff matrices A and B are Return on Equity (ROE) and 1/(1+WACC), respectively. An essential part of the problem formulation is the inclusion of an additional matrix C, composed of the equity capital shares Ke of the companys C_1, C_2, ..., C_n included in the sample and fixed for specific periods P_1, P_2, ..., P_n. Applying traditional methods to find the Nash equilibrium allows us to propose optimal proportions X*, Y* for mixing the players' strategies and determining a balanced capital structure $K_e^N = X^* C^* Y^*$, where C* is a representative fragment of matrix C. The Stackelberg and Germeier equilibria in hierarchical games are found in pure strategies, establishing the following inequality chain: $K_e^N < K_e^{St} < K_e^G$, where the last two values represent the equilibrium equity shares according to Stackelberg and Germeier, respectively. The paper attempts to account for behavioral aspects in capital structure formation, demonstrating how different forms of corporate governance - compromise, leadership, dictation - lead to different relationships between return on equity (ROE), cost of capital (WACC), and its structure.*

Keywords: **bimatrix games, hierarchical games, capital structure, Nash equilibrium, Stackelberg equilibrium, Germeier equilibrium, behavioral aspects.**

1. Introduction

The main controversial issues in the field of corporate finance are the problem of finding the optimal capital structure, in other words, the problem of choosing between debt and equity, as well as the issue of ensuring an acceptable level of return on invested capital. From this perspective, two problematic areas can be identified. The first is the area of corporate finance, including capital structure; the second is the area of expected utility, i.e., return on invested capital. Decisions in the first area are made by company managers, while decisions in the second are made by owners. Modern literature [1] suggests that managers, when faced with a choice between financing sources, will strive for an optimal capital structure. They can increase debt, following trade-off theory, and take advantage of lower borrowing costs, or, conversely, reduce debt, following agency theory. The optimal policy here is one of low leverage, moderate cash balances, and high stock payouts [2].The application of a behavioral approach [3] questions the exclusive rationality of behavior in capital markets and acknowledges conflicts of interest between different groups of economic agents (managers or owners), along with an analysis of their possible causes and consequences for the parties [4;5]. However, no specific solutions capable of satisfying all parties are offered here.

Existing research in the game-theoretic direction originates from the work of the American researcher S.W. Ryu (1985), who considered a model of a non-cooperative game characterizing the conflict of interest between a company's owners and creditors and provided sufficient conditions for finding a Pareto-optimal solution to the corresponding bimatrix game [6]. Ryu's work stimulated increased interest in applying game theory to finance (see review [7]), but until the early 2010s, no models based on conflicts of interest between company owners and managers appear to have been proposed. The situation changed in 2014 with the publication of B. Tudose [8], who formalized the conflict situation as an antagonistic (matrix) game and reduced it to a linear optimization problem. Tudose's approach was continued by the authors in a note [9], which proposed a bimatrix game model and a method for finding an equilibrium (Nash) capital structure in segment S, consisting of five industrial companys which financial and economic indicators were observed over five years. It was assumed that the payoff of Player I (the owner) is Return on Equity (ROE), and that of Player II (the managers) is the inverse of the Weighted Average Cost of Capital (1/(1+WACC)), and that the players do not exchange information with each other but play "each for themselves." This view of the situation can be meaningfully supplemented by the assumption of a possible hierarchical structure in the owner-manager relationship, where the first player is considered as the leader and the second as the follower. The equilibria arising from this formulation are called Stackelberg equilibrium (if the players act as partners at the same hierarchical level, but the leader has the right to make a move first) and Germeier equilibrium (if the leading player is at the top of the hierarchy and informs the lower-level player (agent) of the dependence of his action on the agent's action) [10, ch.6; 11, ch.7].

The objective of game-theoretic modeling in our case is to determine the equilibrium (in the sense of Nash, Stackelberg, and Germeier) values of ROE, WACC, and capital structure, which will allow ones to determine recommended boundaries for their possible variation within the selected segment.

2. Methods

We assume there is a sample of n companies from the same industrial segment C_1, C_2, ..., C_n, belonging to a single owner[1] and managed from a single center. Then, the game matrix A of the first player (a generalized hypothetical owner) can be defined as a matrix consisting of ROE_{ij} values observed for companies Π_i over m years. The corresponding matrix B of the second player (a generalized hypothetical manager) will consist of elements $\boldsymbol{b_{ij}}$, $i=1,...,n; j=1,..,m,$ calculated by the formula [12]

$$\boldsymbol{b_{ij}} = \frac{\mathbf{1}}{\mathbf{1+WACC_{ij}}}. \qquad (1)$$

WACC is calculated using the formula

$$\text{WACC} = R_d \cdot K_d(1-t) + R_e \cdot K_e, \qquad (2)$$

where K_d is the share of debt capital, K_e is the share of equity capital, R_d is the cost of debt capital, R_e is the cost of equity capital, t is the corporate income tax rate (details for determining R_d see in [13]).

Unlike [9], we will consider the strategies of Player I as his choice of a specific company from segment S (rows of matrix A), and the strategies of Player II as his choice of a specific year (columns of matrix B). We will determine the Nash, Stackelberg, and Germeier equilibria in a standard manner, following [14; 11]. In doing so, in addition to matrices A

[1] Examples include private industrial holdings such as the ArcelorMittal, Nippon Steel Corporation, POSCO, Baowu Steel Group etc.

and *B*, used to find the outcome of a particular type of game, it is also necessary to have matrix *C* of equity shares in the capital structure.

2.1.Determination of payoff matrices.

As an empirical base, we will use financial statement data of six large metallurgical companies over six years[2], based on which we determine Return on Equity *ROE* (Table 1).

Table 1. Return on equity (ROE) of six metallurgical company[3]

Company	Periods					
	P1	P2	P3	P4	P5	P6
C1	0.23	0.21	0.6	0.12	0.19	0.15
C2	0.28	0.24	0.78	0.27	0.24	0.09
C3	0.59	0.56	1.01	0.36	0.27	0.26
C4	0.48	0.44	0.77	-0.15	0.09	0.11
C5	0.38	0.36	0.43	0.35	0.91	0.32
C6	0.2	0.17	1.14	1.82	-1.15	

Thus, the payoff matrix *A* will have the form

$$A = \begin{pmatrix} 0.23 & 0.21 & 0.6 & 0.12 & 0.19 & 0.15 \\ 0.28 & 0.24 & 0.78 & 0.27 & 0.24 & 0.09 \\ 0.59 & 0.56 & 1.01 & 0.36 & 0.27 & 0.26 \\ 0.48 & 0.44 & 0.77 & -0.15 & 0.09 & 0.11 \\ 0.38 & 0.36 & 0.43 & 0.35 & 0.91 & 0.32 \\ 0.2 & 0.17 & 1.14 & 1.82 & -1.15 & \end{pmatrix} \quad (3)$$

Using information on dividend payments for the analyzed companies $C_1, C_2, ..., C_6$, we can calculate R_e and the Weighted Average Cost of Capital WACC applying formula (2) (see Table 2).

Table 2. Weighted average cost of capital (WACC) of six metallurgical company

	P1	P2	P3	P4	P5	P6
C1	0.07	0.043	0.071	0.024	0.055	0.087
C2	0.065	0.05	0.082	0.02	0.107	0.0231
C3	0.052	0.045	0.052	0.025	0.086	0.0708
C4	0.022	0.018	0.019	0.025	0.037	0.0525
C5	0.037	0.063	0.0459	0.0548	0.0641	0.0768
C6	0.033	0.044	0.076	0.089	0.054	

Now, exploiting formula (1), we compute the elements of the payoff matrix based on the information in Table 2. As a result, we obtain the payoff matrix

[2] It's important here that the number of firms included in the sample equals the number of observation periods. This allows players to choose from an equal number of pure strategies, putting them on a roughly same footing before the game begins.

[3] The empty cell at location a_{66} in matrix A indicates that it was not possible to calculate ROE for company C_6 in period P_6 due to its liquidation or acquisition.

$$B = \begin{pmatrix} 0.935 & 0.959 & 0.934 & 0.977 & 0.948 & 0.92 \\ 0.939 & 0.952 & 0.924 & 0.98 & 0.903 & 0.977 \\ 0.951 & 0.957 & 0.951 & 0.976 & 0.921 & 0.934 \\ 0.978 & 0.982 & 0.981 & 0.976 & 0.964 & 0.95 \\ 0.964 & 0.941 & 0.956 & 0.948 & 0.94 & 0.929 \\ 0.968 & 0.958 & 0.929 & 0.918 & 0.949 & 0.944 \end{pmatrix} \tag{4}$$

3. Results

3.1. Nash equilibrium capital structure

We will perform a reduction of matrix A based on the dominance principle. Since the elements of row A3 exceed the same elements of rows A1, A2, A4, Player 1 will not choose them, and matrix A is reduced to the form

$$A_r = \begin{pmatrix} 0.59 & 0.56 & 1.01 & 0.36 & 0.27 & 0.26 \\ 0.38 & 0.36 & 0.43 & 0.35 & 0.91 & 0.32 \\ 0.2 & 0.17 & 1.14 & 1.82 & -1.15 & \end{pmatrix}$$

Let's fill the empty space in cell b_{66} of matrix B by the average for the last row[4] (we get b_{66}=0.944). The principle of strategy dominance for the columns of the completed matrix B, due to the inequalities $B_1>B_3$, $B_1>B_5$, $B_2>B_6$, allows us to exclude strategies B_6, B_3, B_5 from consideration. After reduction, we get the matrix B_r of the form

$$B_r = \begin{pmatrix} 0.935 & 0.959 & 0.977 \\ 0.939 & 0.952 & 0.98 \\ 0.951 & 0.957 & 0.976 \\ 0.978 & 0.982 & 0.976 \\ 0.964 & 0.941 & 0.948 \\ 0.968 & 0.958 & 0.918 \end{pmatrix}.$$

Because players I and II can determine which strategies are obviously disadvantageous to their opponents, the matrices A_r and B_r are reduced to 3x3 matrices:

$$\overline{A_r} = \begin{pmatrix} 0.59 & 0.56 & 0.36 \\ 0.38 & 0.36 & 0.35 \\ 0.2 & 0.17 & 1.82 \end{pmatrix}, \overline{B_r} = \begin{pmatrix} 0.951 & 0.957 & 0.976 \\ 0.964 & 0.941 & 0.948 \\ 0.968 & 0.958 & 0.918 \end{pmatrix},$$

which can be reduced repeatedly.

Since for the matrix $\overline{A_r}$ the elements of row № 1 are greater than the same elements of row № 2, the second row can be excluded from consideration. Then the (classical) bimatrix game reduces to a game with matrices

$$\bar{\bar{A}}_r = \begin{pmatrix} 0.59 & 0.56 & 0.36 \\ 0.2 & 0.17 & 1.82 \end{pmatrix}, \bar{\bar{B}}_r = \begin{pmatrix} 0.951 & 0.957 & 0.976 \\ 0.968 & 0.958 & 0.918 \end{pmatrix}. \tag{5}$$

As the positions of the maximal column elements of the matrix $A_1 = \bar{\bar{A}}_r$ do not coincide with the positions of the maximal row elements of the matrix $B_1 = \bar{\bar{B}}_r$ (the best responses of one player to the pure strategies of another player do the same), there is no pure-strategy Nash equilibrium here[5].

To find the mixed Nash equilibrium, we denote the optimal mixed strategy of player I by $X^* = (p, 1-p)$, where p is the probability of player I choosing the first row in matrix A_1, and the optimal mixed strategy of player II by $Y^* = (q_1, q_2, q_3)$, where q_j is the probability of player II choosing the j-th column in matrix B_1, $j=1,2,3$. We will rely on the fact that in an equilibrium situation, players will use strategies such that each pure strategy,

[4] For other methods of filling in data gaps, see, for example, [15].

[5] In the work [9], on the contrary, the Nash equilibrium in pure strategies took place.

played with positive probability, gives the same and maximum expected gain against the opponent's strategy (see, for example, [16, § 2.1]).

To systematize the search through all possible submatrices of matrices A_1, B_1, we make an assumption about the spectrum of player II. In the equilibrium situation, the following mixed strategy choices are available:

1. All three strategies play with positive probability $(q_1>0, q_2>0, q_3>0)$.
2. Two strategies play with positive probability (three variants: {1,2}, {1,3}, {2,3}).

Let $F_1(i,q)$ denote player I's payoff when choosing the *i*-th row, given that player II's strategy profile is $q=(q_1,q_2,q_3)$. Regardless of his strategy profile, player I should receive the same payoff [17, § 3.3] when choosing pure strategies:

$$F_1(1,q)= F_1(2,q) \quad (6)$$

This gives the following equation for q_i $(i=1,2,3)$:

$$(a_{11} - a_{21})q_1 + (a_{12} - a_{22})q_2 + (a_{13} - a_{23})q_3 = 0, \quad (7)$$

where a_{ij} are the components of payoff matrix A1, to which we must add the condition of non-negativity of q_i and the equality

$$q_1 + q_2 + q_3 = 1. \quad (8)$$

If only one of the rows is played, then, according to the complementary slackness theorem [18, § 3.5], condition (6) must be replaced by the inequality

$$F_1(1,q)\geq F_1(2,q), \quad (9),$$

if player I chooses the first pure strategy, and

$$F_1(1,q)\leq F_1(2,q), \quad (10),$$

if one choose the second pure strategy.

Let's move on to formulating the conditions for the second player.

Denoting his payoff for choosing the *j*-th pure strategy by $F_2(p,j)$, where p is the probability of player I choosing row № 1, we have

$$F_2(p,j)=pb_{1j}+(1-p)b_{2j} \quad (11)$$

where b_{ij} are the elements of the payoff matrix B_1.

If the second player's mixed strategy spectrum contains exactly k strategies (k = 1, 2, or 3), then the payoffs for all these strategies must be equal and no less than the payoffs for the remaining strategies.

Let's turn to enumerating the possible strategies for player II. We will consider three cases:

Case A: The second player plays all three strategies with positive probability $(q_1>0, q_2>0, q_3>0)$

Then:

$$F_2(p, 1) = F_2(p, 2) = F_2(p, 3) \quad (12)$$

From these equalities, we obtain two linear equations in p:

$$p*b_{11} + (1-p)*b_{21} = p*b_{12} + (1-p)*b_{22} \quad (13)$$

$$p*b_{12} + (1-p)*b_{22} = p*b_{13} + (1-p)*b_{23}$$

If these equations yield the same p (or the second is a consequence of the first), we solve one of the equations (for example, (13)), from which we get[6]

$$p = (b_{22} - b_{21}) / (b_{11} - b_{12} + b_{22} - b_{21}).$$

Then, from the condition for the first player (equations (7)-(8)) we find q_1, q_2, q_3. and check that all q_j are positive.

Case B: The second player plays exactly two strategies with positive probability.

Consider the subcase: $q_1>0, q_2>0, q_3=0.$

Then the following ratios holds:

1. $F_2(p, 1) = F_2(p, 2)$ (equality for active strategies).
2. $F_2(p, 3) \leq F_2(p, 1)$ (inequality for inactive strategy № 3).

[6] The resulting *p* must be between zero and one.

3. For the first player: if they mix both rows (0<p<1), then $F_1(1, q) = F_1(2, q)$, but now $q_3=0$, therefore:

$(a_{11} - a_{21})q1 + (a_{12} - a_{22})q_2 = 0$;

$q_1 + q_2 = 1$.

From this, we find q1 and q_2, then check $q_1>0$, $q_2>0$.

4. From $F_2(p,1) = F_2(p,2)$, we find p, then check that $0 \le p \le 1$.

5. We check the inequality $F_2(p,3) \le F_2(p,1)$.

We similarly check the other pairs of strategies for the second player: {1,3} and {2,3}.

Case C: Player I plays a pure strategy (p=0 or p=1).

Here, for Player II, the problem is reduced to choosing the best response to Player I's pure strategy. However, if there are several best responses, Player II can mix them with any probabilities. In this case, the optimality condition for Player I's strategy must be verified: if Player I plays p=1, then $F_1(1, q) \ge F_1(2, q)$ for the determined *q*.

Thus, we have the following solution scheme for a 2x3 bimatrix game in mixed strategies [17, 100-101]

1. Suppose Player I mixes both rows (0<p<1). Then, we fix equation (7) for the components *q*.

2. We sort out over Player II's strategies:

o If Player II plays three strategies, then we find p from system (12);

o From system (7)-(8), we find a family of solutions *q*; then check $q_j>0$.

If player II plays two strategies, then for each pair of columns *{j,k}* we set the remaining probabilities equal to zero; from the condition $F_2(p,j)=F_2(p,k)$ we find *p*; from the condition of equality of the first player's payoffs (equation (7) with zero probabilities of the inactive columns), we find q_j,q_k.. At last we check the inequalities for the second player's inactive columns.

3. Suppose the first player plays a pure strategy (p=0 or 1): we find the second player's best responses and check the optimality condition for the first player.

Applying this scheme to solving problems with matrices (4), (5), we obtain a unique Nash equilibrium in mixed strategies, corresponding to the case when player II chooses only the last two columns of matrix B_1 (i.e., prefers periods P_2, P_4 to others), and player I mixes both of her strategies (i.e., chooses companies C_3, C_6 as base ones) with non-zero probabilities. The system of equations (7)–(8) here takes the form

$$\begin{cases} 0.56q_2 + 0.36q_3 = 0.17q_2 + 1.82q_3 \\ q_2 + q_3 = 1 \end{cases}$$

where $q_2 = \frac{146}{185}$; $q_3 = \frac{39}{185}$.

Since player II's choice of active columns must yield the same result, we have the condition $F_2(p,2) = F_2(p,3)$ for finding p. The last one gives the equation

-0.001p + 0.958 = 0.058p + 0.918,

which solution is $p = \frac{40}{59}$. It remains to test the conditions for the inactive column of the form

$F_2(p,1) \le F_2(p,2)$; $F_2(p,1) \le F_2(p,3)$, (14)

as well as the validity of equality (6) for the determined q_j. Testing shows that both inequalities (14) and equality (6) are true.

Let us now calculate the players' winnings using the formulas $F_1^* = F_1(1,\{0,\frac{146}{185};\frac{39}{185}\})$;$F_2^* = F_2(\frac{40}{59},1)$. Considering the well-known formulas of bimatrix game theory (see, for example, [19, § 3.2]),

$U_1^* = X^*\hat{A}Y^*, U_2^* = X^*\hat{B}Y^*$, (15)

where $\hat{A}, \hat{B}$ are the reduced matrices $\bar{\bar{A}}_r, \bar{\bar{B}}_r$ in which the first column is missing, we obtain

$F_1^* \approx 0.5178$ (ROE≈ 0.5178); $F_2^* \approx 0.95732$ (WACC$\approx \frac{1}{0.95732} - 1 \approx 0.0446$).

Now we can find the equilibrium capital structure corresponding to the found ROE and WACC values using a formula similar to (15) [9]

$$K^* = X^*CY^*,$$

where $X^* = \left(\frac{40}{59}, \frac{19}{59}\right)$ is the vector of probabilities for selecting companies C_3, C_6;

$Y^* = \begin{pmatrix} \frac{146}{185} \\ \frac{39}{185} \end{pmatrix}$ is the vector of probabilities for selecting periods from the set P_2, P_4;

C is the "capital structure" matrix, consisting of the shares of equity capital fixed for companies C_3, C_6 in periods P_2, P_4 (see Table 3)

$$C = \begin{pmatrix} 0.4 & 0.58 \\ 0.65 & 0.11 \end{pmatrix}.$$

Table 3. Share of equity capital of companies in the selected segment

	P1	P2	P3	P4	P5	P6
C1	0.62	0.57	0.63	0.7	0.7	0.75
C2	0.56	0.45	0.53	0.5	0.7	0.65
C3	0.36	0.4	0.42	0.58	0.61	0.55
C4	0.58	0.41	0.513	0.44	0.4	0.37
C5	0.13	0.14	0.05	0.05	0.09	0.1
C6	0.42	0.65	0.29	0.11	0.01	-

So, we receive

$$K_e^* = \left(\frac{40}{59}, \frac{19}{59}\right) \cdot \begin{pmatrix} 0.4 & 0.58 \\ 0.65 & 0.11 \end{pmatrix} \cdot \begin{pmatrix} \frac{146}{185} \\ \frac{39}{185} \end{pmatrix} \approx 0.388.$$

Thus, the individual rationality of the owner and manager, as equal participants in a bimatrix game, leads to the following Nash equilibrium capital structure: 38.8% equity and 61.2% debt. The expected return on equity will be 51.78%, and the weighted average cost of total capital will be 4.46%.

If the dimensions of the reduced matrices $\bar{\bar{A}}_r$, $\bar{\bar{B}}_r$ are greater than [2x3] or [3x2], it is advisable to move to the linear complementarity problem and solve it using the Lemke-Hawson algorithm [16, § 2.3].

3.2. Equilibrium capital structure in hierarchical games

3.2.1. Stackelberg equilibrium

Let's first consider a variant in which the players act as partners at the same hierarchical level, but the leader is granted the right of first move. Assuming that the leader is the owner, after choosing strategy $i \in I$ (one of the companies in the C_i sample), it is communicated to the follower (the manager). The follower then chooses strategy $j \in J$ (one of the observation periods P_j), already knowing i. It is convenient to assume that the second player uses mapping strategies of the form $g: I \rightarrow J$,which are functions of the reactions to the leader's strategy. If we denote the set of such strategies by {g}, then the game Γ_1 will be defined as the set $\{I, \{g\}, F(i,g), G(i,g)\}$, $F(i,g)\}$, where $F(i,g) \stackrel{\text{def}}{=} F\big(i, g(i)\big)$, $G(i,g) \stackrel{\text{def}}{=} G(i, g(i))$ – are the payoff functions of players I and II, respectively, in the "base" game, where the second player's strategy is replaced by the function of the second player's reaction to the first player's strategy [11, § 7.1]. Let us find the best guaranteed result F_1 of the leading player in this game. Since all players pursue the goal of maximizing their own payoff, the follower, knowing i, chooses

$$j \in J(i) = Arg \max_{j \in J} G(i,j)$$

– a strategy from the set of best responses to strategy i. The first player knows the payoff function of the second player, as well as the fact that the follower will choose a strategy from the set $J(i)$. In order to evaluate the effectiveness of strategy i, the leading player must find his smallest payoff when the follower implements what he considers to be the best response to this strategy: $W(i) = \min_{j \in J(i)} F(i,j)$ (an estimate of the effectiveness of strategy i). Then the best guaranteed result of the leading player is the maximum payoff he can achieve by controlling his (pure) strategies. Thus, $F_1 = \max_{i \in I} W(i)$. Following [20, Lecture 11], we agree that the second player is benevolent towards the first one, that is, when choosing strategy j, he considers the wishes of the first player:

$$j \in J^*(i) = Arg \max_{j \in J(i)} F(i,j). \qquad (16)$$

In this case, the first player's guaranteed win will be equal to

$$W^*(i) = \max_{j \in J^*(i)} F(i,j),$$

and the best guaranteed win is determined as follows

$$F_1^* = \max_{i \in I} W^*(i).$$

At the same time, a couple of strategies (i_0,j_0) will be a Stackelberg equilibrium [21, § 3.6] if

$$W^*(i_0) = F_1^*(i_0, j_0), \qquad j_0 \in J^*(i_0).$$

Let's find the Stackelberg equilibrium in the game Γ_1 with game matrices (3), (4). To do this, we first define the set $J(i), i=1,\dots,6,$ using the formula (see (16))

$$J(i) = Arg \max_{1 \le j \le 6} b_{ij}.$$

We get $J(1) = J(2) = J(3) = 4$ (the maximum element in rows № 1-3 is at the intersection with column №. 4), $J(4) = 2, J(5) = J(6) = 1$ (the maximum element in row № 4 is in the second column, the maximum element in rows № 5-6 are at the intersection with column № 1).

Finding the guaranteed payoff of player I for all his pure strategies using the formula $W^*(i) = \max_{j \in J^*(i)} a_{ij}$.. As in our case $J^*(i)=J(i)$,

we deduce

$$W^*(1) = a_{14} = 0.12, W^*(2) = a_{24} = 0.27, W^*(3) = a_{34} = 0.36;$$
$$W^*(4) = a_{42} = \mathbf{0.44}, W^*(5) = a_{51} = 0.38, W^*(6) = a_{61} = 0.2.$$

That's why

$$\max_{i \in I} W^*(i) = 0{,}44 = F_1^*.$$

Optimal strategies: $i_0 = 4, j_0 = 2$.

Thus, the Stackelberg equilibrium value of ROE is 44%, which corresponds to a WACC of 1.8% (Table 2) and a share of equity capital of 41%. This situation was observed in period P_2 for company C_4.

3.2.2. Germeier's equilibrium

Let us now turn to the case where the leading player I (the owner) is at the top of the hierarchy and communicates to the lower-level player II (the manager) the dependence of his action on the manager's action. We will assume that player I has complete information about the follower *j's* strategy before choosing strategy *i*. He moves first and communicates to player II a strategy of the form $f: J \to I$ (this version of the hierarchical game will be called, following [11,§ 7.1], game Γ_2. Let us find an expression for the best guaranteed outcome F_2 of the first player in this game. Suppose that the follower, knowing f, chooses j from the set $J(f) = Arg\max_{j \in J} G(f(j), j)$ – set of those strategies that maximize his winnings under the condition that the leading player, knowing strategy j, will play $i = f(j)$.

In the given assumptions, the second player chooses $j \in J(f)$ and the efficiency assessment of strategy f for the leading player is given by the formula

$W(f) = \min_{j \in J(f)} F(f(j), j).$

The best guaranteed outcome for the first player is the maximin of the set of efficiency estimates of the strategies available to him. It has the form

$F_2 = \max_{f \in \{f\}} \min_{j \in J(f)} F(f(j), j).$

To find the best guaranteed result for the leading player, as shown by Yu. B. Germeier [22], it is sufficient to perform optimization over sets I and J. To formulate the corresponding theorem, it is necessary to introduce a number of notations [23]:

- $G_2 = \max_{j \in J} \min_{i \in I} G(i, j)$ – the second player's best guaranteed outcome, given that the first player employs a "punishment" strategy that dictates a response to any strategy $j \in J$ of the second player that minimizes his payoff;
- $E = Arg\max_{j \in J} \min_{i \in I} G(i, j)$ – is the set of maximin strategies of the second player;
- $D_2 = \{(i, j) \in I \times J | G(i, j) > G_2\}$ - is the set of situations in the game in which the second player can obtain more than the best guaranteed outcome;
- $M = \min_{j \in E} \max_{i \in I} F(i, j)$ – is the first player's guaranteed outcome;
- $K = \begin{cases} \max_{(i,j)} F(i, j), & D_2 \neq \emptyset \\ -\infty, & D_2 = \emptyset \end{cases}.$

The following theorem is valid (Yu.B. Germeier)

Theorem 1. The best guaranteed outcome in the game Γ_2 is equal to

$$F_2 = \max\{K, M\}.$$

Let's turn to defining the Germeier equilibrium, in accordance with Theorem 1. We begin by searching for the best guaranteed outcome for the second player, provided that the first player uses a "punishment" strategy (that is, sets itself the task of maximizing the WACC[7] rather than ROE). By analogy with matrix games, we denote β_i the value equal to $\min_j b_{ij}$ (the minimum along the column of matrix B), then

$\beta_1 = \min_i b_{i1} = 0.935; \beta_2 = \min_i b_{i2} = 0.941; \beta_3 = \min_i b_{i3} = 0.924;$

$\beta_4 = \min_i b_{i4} = 0.918; \beta_5 = \min_i b_{i5} = 0.903; \beta_6 = \min_i b_{i6} = 0.902.$

Thus, the guaranteed result of the follower (manager) is equal to $G_2 = \max_j \beta_j = 0.941$. The strategy that provides the follower with this result forms a set $E = Arg\max_{1 \le j \le 6} \min_{1 \le j \le 6} b_{ij} = 2$ (second column).

Now let's define the set $D_2 = \{(i, j) | b_{ij} > G_2\}$. It consists of twenty two pairs:

$D_2 = \{(1,2), (1,4), (1,5); (2,2), (2,4), (2,6); (3,1), (3,2), (3,3), (3,4); (4,1), (4,2), (4,3), (4,4)(4,5), (4,6); (5,1), (5,3), (5,4); (6,1), (6,2), (6,5)\}.$

It is easy to find the leading player's greatest payoff in these situations: $K = \max_{(i,j) \in D_2} a_{ij} = 0.77$. Application of the minimax principle gives $M = \min_{j=2} \left(\max_i a_{ij} \right) = \max_i a_{i2} = 0.56$. As a result, the payoff of player I is max{0.77,0.56}=0.77, The

[7] For example, the owner can influence dividend policy, initiate risky projects, and block refinancing.

equilibrium game strategy is to choose company C_4 and period P_3. The corresponding (Germeier-balanced) ROE is 77%, the WACC is 1.9%, and the equity ratio is K_e = 51.3%.

It should be noted that in the hierarchical game Γ_1, the owner and manager can swap roles, with the manager seeking to minimize the WACC having the first move. This is not possible in the game Γ_2: the manager's strategy of punishing the owner is not applicable.

4. Conclusion

Comparing the results obtained in a situation where the owner and manager act independently (Nash equilibrium), without coordinating their decisions, the equilibrium capital structure consists of 38.8% equity and 61.2% debt. The expected return on equity (ROE) is 51.78%. The owner seeks to maximize ROE by exploiting the effect of financial leverage, while the manager, striving for a low WACC, implicitly accepts a relatively high proportion of debt.

In a situation where the owner acts as the first player (Stackelberg equilibrium), the capital structure shifts toward greater conservatism. The proportion of equity increases to 41%, while ROE decreases to 44%. This is explained by the fact that the number of available pure strategies in the Stackelberg equilibrium is smaller than the set of situations in the mixed Nash equilibrium. This results in a safer, but less profitable, capital structure.

A rigid hierarchy with a coercion strategy (Germeier equilibrium) allows the owner to choose the most advantageous situation, where ROE = 77% and the equity ratio is 51.3%, which corresponds to the most conservative financial structure. The paradoxical combination of a high equity ratio and high profitability is explained by the owner, using the threat of "punishment," selecting from real data a historical period and a company where the company demonstrated exceptionally high operating efficiency, which allowed for a high ROE even with low leverage. However, this equilibrium is based on the assumption of complete power and awareness of the owner, which is rarely achievable in practice.

The conducted game-theoretic modeling shows that capital structure optimization cannot be considered in isolation from the governance model and the behavioral characteristics of the participants. Thus, with decentralized governance (Nash equilibrium), conflicts of interest predominate, leading to risky compromises. With formal leadership (according to Stackelberg), a situation can arise that worsens the outcome for everyone. With a hierarchy with coercion mechanism (according to Germeier), it is possible to minimize agency conflict and realize a potentially efficient state, but at the cost of high demands on the owner's awareness and power.

The obtained results can be used to organize the simulation modeling process in order to determine possible options for changing ROE, WACC, and Ke by recalculating the equilibrium values of these quantities using the presented game-theoretic algorithms based on extended matrices A, B, C with added rows of values predicted for the future period (see, for example, [24]).

The behavioral aspects identified during the game-theoretic modeling allow us to conclude that the issue of choosing a capital structure is not only about minimizing WACC and maximizing ROE, but also the problem of constructing a corporate governance system that, taking into account the behavioral preferences of the owner and manager, will minimize agency costs and guide the company toward a financially stable and profitable equilibrium. The presented game-theoretic models, supplemented by behavioral aspects, provide a powerful tool for decision-making in conditions of conflicting interests and imperfect rationality of participants [3, p. 94].

As a possible continuing this theme, we can point to a cooperative two-person game-theoretic model (one step in this direction was made in [25]), as well as trimatrix game models with uncertainty, where the stock market acts as the third player. More complex options, allowing us to move on to the study of the stability of equilibrium states, involve the transition from discrete models to continuous ones (differential games [26, chap.7], [21]), but this is a topic for a separate discussion.

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