# VERTEX ANTIMAGIC EDGE SLITHER LABELING OF MIRROR LAGOON STEP GRAPH

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**Abstract:** The step graph are graphs that help us to understand and comprehend different situations in real life. These graphs are easy to construct and studying these graphs are far too easy to apprehend. In this paper, an attempt has been made to apply the vertex antimagic edge slither labeling to Lagoon step graph.

**Keywords**: Antimagic labeling, antimagic slither labeling, lagoon step graph, Mirror Lagoon Step Graph

**Introduction**: The labeling in graphs play a vital role in studying many real life conditions. In modern day, the graph theoretic scholars and researchers have come up with a variety of labeling and have proved different properties. These labeling structures juxtaposed with various other graph theoretic concepts have paved way for different dimension in the study of graph theory.

**Antimagic Labeling**: The idea of labeling in graph refers to the assignment of positive integers to the vertices or edges or both. Antimagic magic labeling was first defined by Martin Baca and Mirka Miller [4] whereas the concept of (a, d) antimagic labeling was first given by R. Bodendiek and G. Walther [1]. The definition of (a, d) antimagic labeling in graphs goes as follows; "A graph G is an (a, d) vertex antimagic edge labelled graph if there exists a positive integer $a$ and a non-negative integer $d$ and a bijective map given by the function $\alpha: E \rightarrow \{1, 2, \ldots, |E(G)|\}$ such that the induced mapping $\beta: V(G) \rightarrow W$, where $W = \{a, a+d, a+2d, \ldots, a+|V-1|d\}$is also a bijection".

**Slither Labeling**: The idea of slither labeling is the result of antimagic labeling. The labeling of edges of the graph are done based on the slither formation making sure the graph still is antimagic.

**Lagoon Step Graph**: A lagoon step graph denoted by ($L_m S_n$) is obtained from an L-shaped path on m vertices and step like structure formed from cycle on n vertices. A manipulation of the lagoon step graph is the mirror lagoon step graph ($M L_m S_n$) that is obtained by attaching a lagoon step graph to its mirror image. The number of vertices in the graph is 3m + n – 4 and the number of edges is 3m + n – 3. The mirror lagoon step graph under consideration in this paper is due to J. Vasanthi and N. Ramya [8].

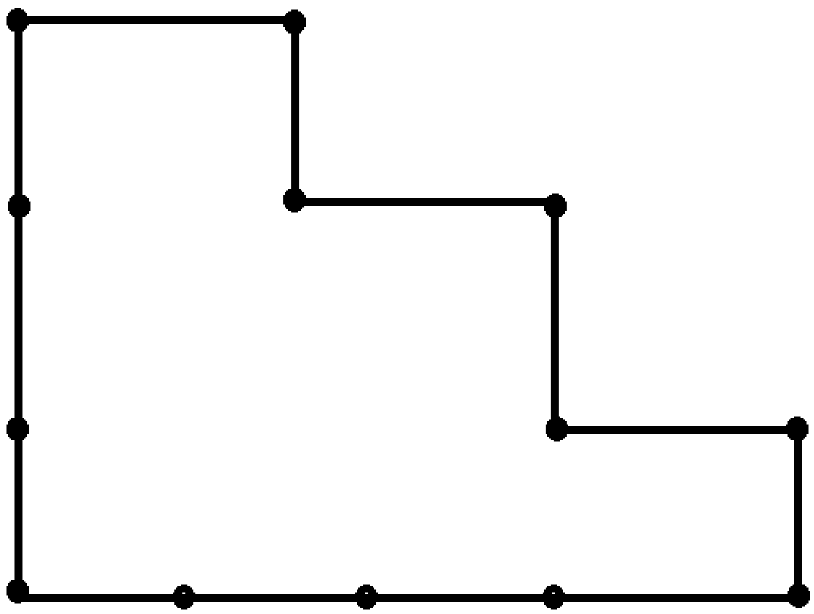

**Fig 1. Lagoon Step Graph $L_3S_3$.**

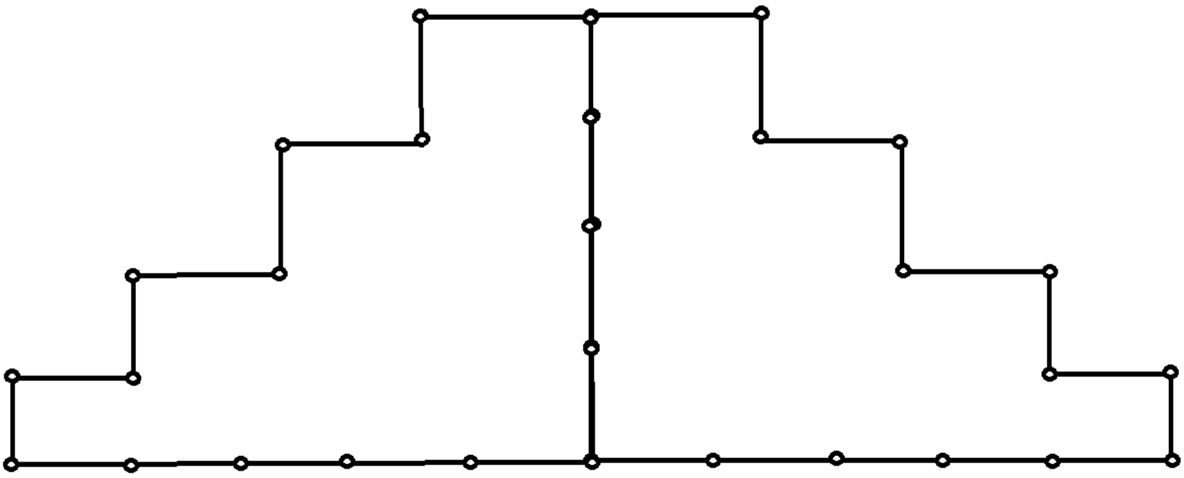

**Fig 2. Mirror Lagoon Step Graph ($ML_9S_4$)**

**Main Result**:

**Theorem**: The Mirror Lagoon Step graph admits vertex antimagic edge slither labeling.

**Proof**: Consider the mirror lagoon step graph given by $M\ L_m\ S_n$, m, n $\geq$ 2. The edge labels of the graph is done in slither pattern with positive integers in such a way that the edges are all different. The label of a vertex is the sum of labels of the edges that are incident with the vertex.

Consider the Mirror Lagoon Step graph $M\ L_m\ S_n$. Let m = 7 and n = 3 be the arbitrary values of m and n. The slither labeling in the graph is given by;

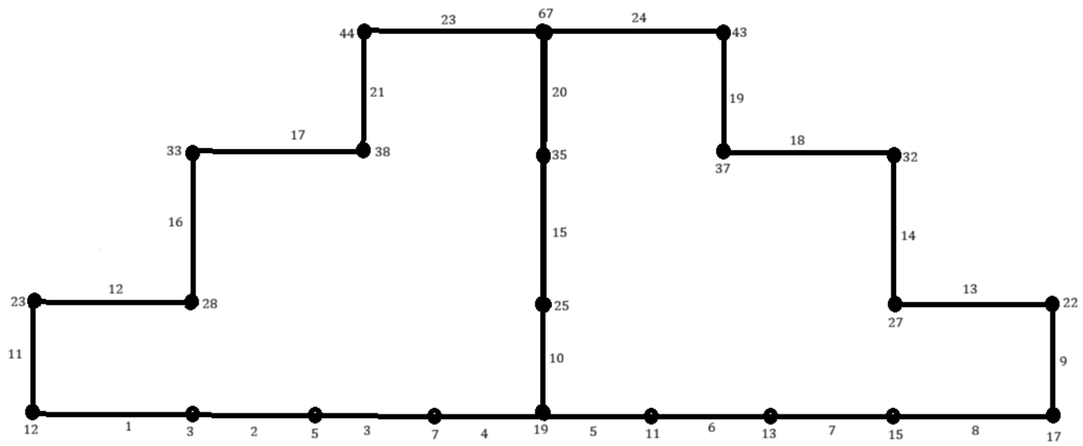


**Fig 3. Vertex Antimagic Edge Slither Labeling in $M\ L_7\ S_3$**

In the above graph, the vertex labels are all distinct and so are the labels of the edges and hence the graph vertex antimagic edge slither labelled for all values of m, n $\geq$ 2.

**Remarks:**

1. The graphs that are k-regular does not admit vertex antimagic edge slither labeling.
2. The Lagoon Step graph is a 2-regular graph and does not admit vertex antimagic edge slither labeling.

**Conclusion**: In this paper, a special type of step graph is considered and the applicability of vertex antimagic edge slither labeling is established. Also it is noted that the k-regular graphs are not vertex antimagic edge slithered. The same concept can be applied to a variety of graphs.

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