

Hypersoft Gpr Closed Sets in Hypersoft Topological Spaces

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Abstract: Smarandache introduced the concept of hypersoft set which is a generalization of soft set. In this paper, we introduce and study a new class of generalized closed set, namely hypersoft generalized preregular (hys gpr for short) closed sets and hypersoft generalized preregular open sets in hypersoft topological spaces. Also we study the separation axioms of hypersoft generalized preregular closed sets, namely hypersoft preregular $T_{1/2}$ space and hypersoft preregular $T^*_{1/2}$ space and their properties are discussed.

Keywords: hypersoft set, hypersoft topological space, hypersoft generalized preregular closed sets, hypersoft generalized preregular open sets, hys pr $T_{1/2}$ space and hys pr $T^*_{1/2}$ space.

Mathematics Subject Classification: 54D10, 54D15.

1 INTRODUCTION

Molodtsov [7] initiated the concept of a soft set theory, which is a completely new approach for modelling vagueness and uncertainty. Maji et al. in [6] developed various operators and basic notions for soft set and conducted a more detailed theoretical analysis of soft set theory.

Shabir and Naz [16] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters and studied some basic notions of soft topological spaces such as soft open and soft closed sets. Later on various authors continued to study the properties of soft topological space.

Abbas, Murtaza and Smarandache[1] were introduced the basic operations on hypersoft sets and hypersoft point Sagvan Y. Musa, Baravan A [8] was developed the concept of hypersoft topological spaces. V.Inthumathi et al.,[4] introduced hypersoft semi open sets in hypersoft topological spaces. Mythili S et al[9,10] introduced and studied the hypersoft α -open, hypersoft preopen, hypersoft semi open, hypersoft sg-closed set, hypersoft gs-closed set, hypersoft αg -closed set, hypersoft rg-closed set and hypersoft $g\beta$ -closed sets, hypersoft generalized closed sets in hypersoft topological spaces. In this direction, introduce and analyze a new class of hypersoft closed set called hypersoft generalized preregular closed sets which are weaker form of the hypersoft closed sets. Also we study the separation axioms of hypersoft generalized preregular closed sets, namely hypersoft preregular $T_{1/2}$ space and hypersoft preregular $T^*_{1/2}$ space in hypersoft topological space.

2 PRELIMINARIES

We recall some basic definitions that are used in the sequel. Throughout the paper, let X be an initial universe, $P(X)$ be the power set of X and $E_1, E_2, \dots, E_n$ the pairwise disjoint sets of parameters. Let A_i, B_i be the non empty subsets of E_i for each $i=1,2,\dots,n$.

Definition 2.1: [18] A hypersoft set can be defined by the pair $(F, A_1 \times A_2 \times \dots \times A_n)$ over X , where a mapping $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(X)$.

Simply, we write the symbol E for $E_1 \times E_2 \times \dots \times E_n$, and A for $A_1 \times A_2 \times \dots \times A_n$ and B for $B_1 \times B_2 \times \dots \times B_n$. Clear, each element in A, B and E is an n -tuple element.

We can represent a hypersoft set (F, A) as an ordered pair,

$$(F, A) = \{(\alpha, F(\alpha)) : \alpha \in A\}$$

Definition 2.2: [13] For any two hypersoft sets (F, A) and (G, B) over a common universe X , we say that (F, A) is a hypersoft subset of (G, B) if

- (i) $A \subseteq B$ and

(ii) $F(\alpha) \subseteq G(\alpha)$ for all $\alpha \in A$.

We write $(F,A) \subseteq (G,B)$.

(F,A) is said to be a hypersoft superset of (G,B) , if (G,B) is a hypersoft subset of (F,A) .

We denote it by $(F,A) \supseteq (G,B)$.

Definition 2.3: [13] Two hypersoft sets (F,A) and (G,B) over a common universe X are said to be hypersoft equal if (F,A) is a hypersoft subset of (G,B) and (G,B) is a hypersoft subset of (F,A) .

Definition 2.4: [13] Let $A = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a set of parameters. The NOT set of A denoted by $\neg A$ is defined by $\neg A = \{\neg\alpha_1, \neg\alpha_2, \dots, \neg\alpha_n\}$ where $\neg\alpha_i = \text{not } \alpha_i$ for $i = 1, 2, \dots, n$.

Proposition 2.5: [13] For any subsets $A, B \subseteq E$.

(i) $\neg(\neg A) = A$.

(ii) $\neg(A \cup B) = \neg A \cap \neg B$.

(iii) $\neg(A \cap B) = \neg A \cup \neg B$.

Definition 2.6: [18] The complement of a hypersoft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$ where $F^c: A \rightarrow P(X)$ is a mapping given by $F^c(\alpha) = X \setminus F(\alpha)$ for all $\alpha \in A$.

Definition 2.7: [18] A hypersoft set (F,A) over X is said to be a relative null hypersoft set, denoted by (Φ, A) , if for all $\alpha \in A$, $F(\alpha) = \emptyset$.

Definition 2.8: [18] A hypersoft set (F,A) over X is said to be a relative whole hypersoft set, denoted by (Ψ, A) , if for all $\alpha \in A$, $F(\alpha) = X$.

Definition 2.9: [18] Difference of two hypersoft sets (F,A) and (G,B) over a common universe X , is a hypersoft set (H,C) , where $C = A \setminus B$ and for all $\alpha \in C$, $H(\alpha) = F(\alpha) \setminus G(\alpha)$. We write $(F,A) \setminus (G,B) = (H,C)$.

Definition 2.10: [18] The union of (F,A) and (G,B) of two hypersoft sets over X , is a hypersoft set $(H,C) = (F,A) \cup (G,B)$ where $C = A \cup B$ and for all $\alpha \in C$, $H(\alpha) = F(\alpha) \cup G(\alpha)$.

Definition 2.11: [18] Intersection of two hypersoft sets (F,A) and (G,B) over a common universe X , is a hypersoft set $(H,C) = (F,A) \cap (G,B)$, where $C = A \cap B$ and for all $\alpha \in C$, $H(\alpha) = F(\alpha) \cap G(\alpha)$.

Definition 2.12: [8] Let τ be a collection of hypersoft sets over X , then τ is said to be a hypersoft topology over X if

(i) $(\Phi, E), (\Psi, E)$ belongs to τ

(ii) the intersection of any two hypersoft sets in τ belongs to τ ,

(iii) the union of any number of hypersoft sets in τ belongs to τ .

Then (X, τ, E) is called a hypersoft topological spaces over X .

Definition 2.13: [8] Let (X, τ, E) be a hypersoft topological space over X , then the members of τ are said to be hypersoft open sets in X .

Definition 2.14: [8] Let (X, τ, E) be a hypersoft topological space over X . A hypersoft set (F,E) over X is said to be a hypersoft closed set in X , if its complement $(F,E)^c$ belongs to τ .

Proposition 2.15: [8] Let (X, τ, E) be a hypersoft topological space over X . Then

(i) $(\Phi, E), (\Psi, E)$ are hypersoft closed set over X ,

(ii) the union of any two hypersoft closed sets is a hypersoft closed set over X ,

(iii) the intersection of any number of hypersoft closed sets is a hypersoft closed set over X .

Definition 2.16: [8] Let (X, τ, E) be a hypersoft topological space over X and (F, E) be a hypersoft set then

- (i) The hypersoft interior of (F, E) is the hypersoft set
 $\text{hint}(F, E) = \bigcup \{(G, E): (G, E) \text{ is the hypersoft open and } (G, E) \subseteq (F, E)\}$
- (ii) The hypersoft closure of (F, E) is the hypersoft set
 $\text{hcl}(F, E) = \bigcap \{(G, E): (G, E) \text{ is the hypersoft closed and } (G, E) \supseteq (F, E)\}$

Proposition 2.17: [8] Let (X, τ, E) be a hypersoft topological space over X and let (F_1, E) and (F_2, E) be two hypersoft sets over X . Then

- (i) $\text{hint}(\Phi, E) = (\Phi, E)$ and $\text{hint}(\Psi, E) = (\Psi, E)$.
- (ii) $\text{hint}(F_1, E) \subseteq (F_1, E)$.
- (iii) $(F_1, E) \subseteq (F_2, E)$ implies $\text{hint}(F_1, E) \subseteq \text{hint}(F_2, E)$.
- (iv) $\text{hint}(F_1, E) \cap \text{hint}(F_2, E) = \text{hint}((F_1, E) \cap (F_2, E))$.
- (v) $\text{hint}(F_1, E) \cup \text{hint}(F_2, E) \subseteq \text{hint}((F_1, E) \cup (F_2, E))$.
- (vi) $\text{hint}(\text{hint}(F_1, E)) = \text{hint}(F_1, E)$.

Proposition 2.18: [8] Let (X, τ, E) be a hypersoft topological space over X and let (F_1, E) and (F_2, E) be two hypersoft sets over X . Then

- (i) $\text{hcl}(\Phi, E) = (\Phi, E)$ and $\text{hcl}(\Psi, E) = (\Psi, E)$.
- (ii) $(F_1, E) \subseteq \text{hcl}(F_1, E)$.
- (iii) $(F_1, E) \subseteq (F_2, E)$ implies $\text{hcl}(F_1, E) \subseteq \text{hcl}(F_2, E)$.
- (iv) $\text{hcl}((F_1, E) \cup (F_2, E)) = \text{hcl}(F_1, E) \cup \text{hcl}(F_2, E)$.
- (v) $\text{hcl}((F_1, E) \cap (F_2, E)) \subseteq \text{hcl}(F_1, E) \cap \text{hcl}(F_2, E)$.
- (vi) $\text{hcl}(\text{hcl}(F_1, E)) = \text{hcl}(F_1, E)$.

Definition 2.19: [10] Let (F, E) be a hypersoft set of a hypersoft topological space (X, τ, E) is said to be

- (i) Hypersoft regular closed if $\text{hcl}(\text{hint}(F, E)) = (F, E)$.
- (ii) Hypersoft regular open if $(F, E) = \text{hint}(\text{hcl}(F, E))$.
- (iii) Hypersoft semi closed if $\text{hint}(\text{hcl}(F, E)) \subseteq (F, E)$.
- (iv) Hypersoft semi open if $(F, E) \subseteq \text{hcl}(\text{hint}(F, E))$.
- (v) Hypersoft pre closed if $\text{hcl}(\text{hint}(F, E)) \subseteq (F, E)$.
- (vi) Hypersoft pre open if $(F, E) \subseteq \text{hint}(\text{hcl}(F, E))$.
- (vii) Hypersoft α closed if $\text{hcl}(\text{hint}(\text{hcl}(F, E))) \subseteq (F, E)$.
- (viii) Hypersoft α open if $(F, E) \subseteq \text{hint}(\text{hcl}(\text{hint}(F, E)))$.

Definition 2.20: [10] Let (X, τ, E) be a hypersoft topological space over X and (F, E) be a hypersoft set then

- (i) The hypersoft pre interior of (F, E) is the hypersoft set
 $\text{hpint}(F, E) = \bigcup \{(G, E): (G, E) \text{ is the hypersoft pre open and } (G, E) \subseteq (F, E)\}$
- (ii) The hypersoft pre closure of (F, E) is the hypersoft set
 $\text{hpcl}(F, E) = \bigcap \{(G, E): (G, E) \text{ is the hypersoft pre closed and } (G, E) \supseteq (F, E)\}$

Definition 2.21: [4] A hypersoft set (F, E) is called a hypersoft generalized closed (hys g-closed) set in a hypersoft topological space (X, τ, E) if $\text{hcl}(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is hypersoft open set in X . A hypersoft set (F, E) is called a hypersoft generalized open (hys g-open) set in a hypersoft topological space (X, τ, E) if the relative complement $(F, E)'$ is hypersoft g-closed set in X .

Definition 2.22: [10] A hypersoft set (F, E) of a hypersoft topological space (X, τ, E) is called

- (i) a hypersoft α -generalized closed (hys α g-closed) set if $h\alpha cl(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is hypersoft open set in X .
- (ii) a hypersoft regular generalized closed (hys rg-closed) set if $hcl(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is hypersoft regular open set in X .

Definition 2.23: [10] A hypersoft set (F, E) is called a hypersoft α -generalized open (hys α g-open) (respectively hypersoft regular generalized open (hys rg open)) set in a hypersoft topological space (X, τ, E) if the relative complement $(F, E)'$ is hypersoft α -closed (respectively hypersoft regular generalized closed) set in X .

3 HYPERSOFT GENERALIZED PRE REGULAR CLOSED SETS

In this section we introduce hypersoft generalized pre regular closed sets in the hypersoft topological space and study some of their properties.

Definition 3.1: A hys set (F, E) of a hypersoft topological space (X, τ, E) is said to be a hypersoft generalized pre regular closed (hys gpr-closed for short) set if $hpcl(F, E) \subseteq (U, E)$ whenever $A \subseteq (U, E)$ and (U, E) is a hys regular open set in (X, τ, E) . The family of all hys gpr-closed sets of a hypersoft topological space (X, τ, E) is denoted by $\text{hys gprc}(X, E)$.

Example 3.2: Let $X = \{x_1, x_2\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$ and $E_3 = \{e_4\}$ where the hypersoft sets over X is defined as follows

$$\begin{aligned} (F_1, E) &= \{((e_1, e_3, e_4), \phi), ((e_2, e_3, e_4), \phi)\} \\ (F_2, E) &= \{((e_1, e_3, e_4), \phi), ((e_2, e_3, e_4), \{x_1\})\} \\ (F_3, E) &= \{((e_1, e_3, e_4), \phi), ((e_2, e_3, e_4), \{x_2\})\} \\ (F_4, E) &= \{((e_1, e_3, e_4), \phi), ((e_2, e_3, e_4), \{x_1, x_2\})\} \\ (F_5, E) &= \{((e_1, e_3, e_4), \{x_1\}), ((e_2, e_3, e_4), \phi)\} \\ (F_6, E) &= \{((e_1, e_3, e_4), \{x_1\}), ((e_2, e_3, e_4), \{x_1\})\} \\ (F_7, E) &= \{((e_1, e_3, e_4), \{x_1\}), ((e_2, e_3, e_4), \{x_2\})\} \\ (F_8, E) &= \{((e_1, e_3, e_4), \{x_1\}), ((e_2, e_3, e_4), \{x_1, x_2\})\} \\ (F_9, E) &= \{((e_1, e_3, e_4), \{x_2\}), ((e_2, e_3, e_4), \phi)\} \\ (F_{10}, E) &= \{((e_1, e_3, e_4), \{x_2\}), ((e_2, e_3, e_4), \{x_1\})\} \\ (F_{11}, E) &= \{((e_1, e_3, e_4), \{x_2\}), ((e_2, e_3, e_4), \{x_2\})\} \\ (F_{12}, E) &= \{((e_1, e_3, e_4), \{x_2\}), ((e_2, e_3, e_4), \{x_1, x_2\})\} \\ (F_{13}, E) &= \{((e_1, e_3, e_4), \{x_1, x_2\}), ((e_2, e_3, e_4), \phi)\} \\ (F_{14}, E) &= \{((e_1, e_3, e_4), \{x_1, x_2\}), ((e_2, e_3, e_4), \{x_1\})\} \\ (F_{15}, E) &= \{((e_1, e_3, e_4), \{x_1, x_2\}), ((e_2, e_3, e_4), \{x_2\})\} \\ (F_{16}, E) &= \{((e_1, e_3, e_4), \{x_1, x_2\}), ((e_2, e_3, e_4), \{x_1, x_2\})\} \\ T &= \{(F_1, E), (F_5, E), (F_7, E), (F_8, E), (F_{16}, E)\} \end{aligned}$$

Then (F, τ, E) is a hypersoft topological space.

The collection of all hys open sets are

$$\{(F_1, E), (F_5, E), (F_7, E), (F_8, E), (F_{16}, E)\}$$

The set of all hys closed sets are

$$\{(F_1, E), (F_9, E), (F_{10}, E), (F_{12}, E), (F_{16}, E)\}$$

The collection of hys pre-open sets are

$$\{(F_1, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_{13}, E), (F_{14}, E), (F_{15}, E), (F_{16}, E)\}$$

The collection of hys pre-closed sets are

$$\{(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_9, E), (F_{10}, E), (F_{11}, E), (F_{12}, E), (F_{16}, E)\}$$

The collection of all hys regular open sets are

$$\{(F_1, E), (F_{16}, E)\}$$

The above collection of all the hys sets are hys gpr closed sets.

Theorem 3.3: Every hys closed set (F, E) in (X, τ, E) is a hys gpr-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys closed set in (X, τ, E) , we have $hcl(F, E) = (F, E)$. Therefore $hpcl(F, E) \subseteq hcl(F, E) = (F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Example 3.4: Consider the hypersoft topological space of Example 3.2., Here (F_2, E) , (F_3, E) , (F_4, E) , (F_5, E) , (F_6, E) , (F_7, E) , (F_8, E) , (F_{11}, E) , (F_{13}, E) , (F_{14}, E) , (F_{15}, E) are hys gpr-closed set but not hys closed set in (X, τ, E) .

Theorem 3.5: Every hys α closed set in (X, τ, E) is an hys gpr-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $A \subseteq (U, E)$. Since (F, E) is $N\alpha CS$ in (X, τ, E) , we have $hcl(hint(hcl(F, E))) \subseteq (F, E)$, now $(F, E) \subseteq hcl(F, E)$, $hcl(hint(F, E)) \subseteq hcl(hint(hcl(F, E))) \subseteq F, E$. Therefore $hpcl(F, E) = (F, E) \cup hcl(hint(F, E)) \subseteq (F, E) \cup (F, E) = (F, E) \subseteq (U, E)$. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Example 3.6: Consider the hypersoft topological space of Example 3.2., Here (F_8, E) is hys gpr-closed set but not hys α closed set in (X, τ, E) .

Theorem 3.7: Every hys p-closed set in (X, τ, E) is an hys gpr-closed set in (X, τ, E) but not conversely

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys p-closed set in (X, τ, E) , we have $hcl(hint(F, E)) \subseteq (F, E)$. Therefore $hpcl(F, E) = (F, E) \cup hcl(hint(F, E)) \subseteq (F, E) \cup (F, E) = (F, E) \subseteq (U, E)$. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Example 3.8: Consider the hypersoft topological space of Example 3.2., Here (F_5, E) , (F_6, E) , (F_7, E) , (F_8, E) , (F_{13}, E) , (F_{14}, E) , (F_{15}, E) are hys gpr-closed set but not hys p-closed set in (X, τ, E) .

Theorem 3.9: Every hys g-closed set in (X, τ, E) is a hys gpr-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys g-closed in (X, τ, E) and every hys regular open set in (X, τ, E) is a hys open set in (X, τ, E) . Therefore $hpcl(F, E) \subseteq hcl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Example 3.10: Consider the hypersoft topological space of Example 3.2., Here (F_5, E) , (F_6, E) , (F_7, E) , (F_8, E) , are hys gpr-closed set but not hys g-closed set in (X, τ, E) .

Theorem 3.11: Every hys αg -closed set in (X, τ, E) is a hys gpr-closed set in (X, τ, E) but not conversely

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys αg -closed set in (X, τ, E) and every hys regular open set in (X, τ, E) is a hys open set in (X, τ, E) . Therefore $hpcl(A) \subseteq hacl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Example 3.12: Consider the hypersoft topological space of Example 3.2., Here (F_8, E) , is hys gpr-closed set but not hys αg -closed set in (X, τ, E) .

Theorem 3.13: Every hys rag-closed set in (X, τ, E) is a hys gpr-closed set in (X, τ, E) but not conversely

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys rag-closed set in (X, τ, E) . Therefore $hpcl(F, E) \subseteq hacl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Theorem 3.14: Every hys gp-closed set in (X, τ, E) is a hys gpr-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys gp-closed set in (X, τ, E) and every hys regular open set in (X, τ, E) is a hys open set in (X, τ, E) . Therefore $hpcl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

Example 3.15: Consider the hypersoft topological space of Example 3.2., Here (F_5, E) , (F_7, E) , (F_8, E) , are hys gpr-closed set but not hys gp-closed set in (X, τ, E) .

Theorem 3.16: Every hys rg-closed set in (X, τ, E) is a hys gpr-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys rg-closed set in (X, τ, E) . Therefore $hpcl(F, E) \subseteq hcl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys gpr-closed set in (X, τ, E) .

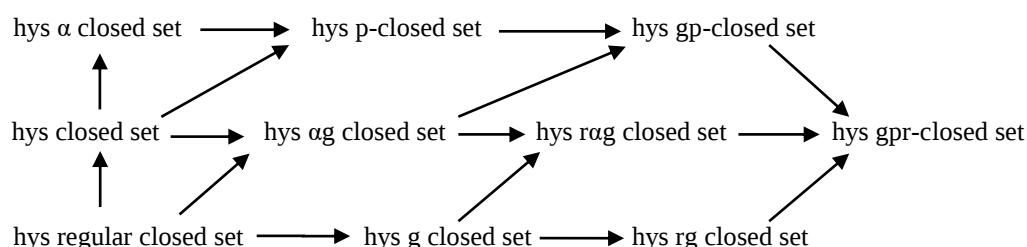
Theorem 3.17: Every hys g-closed set in (X, τ, E) is a hys α g-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys g-closed set in (X, τ, E) . Therefore $haccl(F, E) \subseteq hcl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys α g-closed set in (X, τ, E) .

Theorem 3.18: Every hys g-closed set in (X, τ, E) is a hys rg-closed set in (X, τ, E) but not conversely.

Proof: Let (U, E) be a hys regular open set in (X, τ, E) such that $(F, E) \subseteq (U, E)$. Since (F, E) is hys g-closed set in (X, τ, E) and every hys regular open set in (X, τ, E) is a hys open set in (X, τ, E) . Therefore $hcl(F, E) \subseteq (U, E)$, by hypothesis. Hence (F, E) is a hys rg-closed set in (X, τ, E) .

The following diagram, we have provided the relation between hys gpr-closed set and the other existed NSs.



In this diagram by “ $(F, E) \longrightarrow (G, E)$ ” means (F, E) implies (G, E) but not conversely.

Theorem 3.19: Let (X, τ, E) be a hypersoft topological space. Then for every $(F, E) \in \text{hys gprc}(X, E)$ and for every hys set $(G, E) \in \text{hys}(X, E)$, $(F, E) \subseteq (G, E) \subseteq \text{hpcl}(F, E)$ implies $(G, E) \in \text{hys gprc}(X, E)$.

Proof: Let $(G, E) \subseteq (U, E)$ and (U, E) is a hys regular open set in (X, τ, E) . Since $(F, E) \subseteq (G, E)$, then $(F, E) \subseteq (U, E)$. Given (F, E) is a hys gpr-closed set, it follows that $\text{hpcl}(F, E) \subseteq (U, E)$. Now $(G, E) \subseteq \text{hpcl}(F, E)$ implies $\text{hpcl}(G, E) \subseteq \text{hpcl}(\text{hpcl}(F, E)) = \text{hpcl}(F, E)$. Thus, $\text{hpcl}(G, E) \subseteq (U, E)$. This proves that $(G, E) \in \text{hys gprc}(X, E)$.

Theorem 3.20: If (F, E) is a hys regular open set and a hys gpr-closed set in (X, τ, E) , then (F, E) is a hys p-closed set in (X, τ, E) .

Proof: Since $(F, E) \subseteq (F, E)$ and (F, E) is a hys regular open set in (X, τ, E) , by hypothesis, $\text{hpcl}(F, E) \subseteq (F, E)$. But since $(F, E) \subseteq \text{hpcl}(F, E)$. Therefore $\text{hpcl}(F, E) = (F, E)$. Hence (F, E) is a hys p-closed set in (X, τ, E) .

Definition 3.21: Let (X, τ, E) be a hypersoft topological space and (F, E) hypersoft set of (X, τ, E) . Then

$\text{hgprcl}(F, E) = \cap \{(G, E) : (G, E) \text{ is a hys gpr-closed set in } (X, \tau, E) \text{ and } (F, E) \subseteq (G, E)\}$ and

$\text{hgprint}(F, E) = \cup \{(H, E) : (H, E) \text{ is a hys gpr-open set in } (X, \tau, E) \text{ and } (H, E) \subseteq (F, E)\}$.

Lemma 3.22: Let (F, E) and (G, E) are hypersoft subsets of (X, τ, E) . Then the following results are obvious.

- a) $\text{hgprcl}(0_N) = 0_N$.
- b) $\text{hgprcl}(1_N) = 1_N$.
- c) $(F, E) \subseteq \text{hgprcl}(F, E)$.

$(F, E) \subseteq (G, E) \Rightarrow \text{hgprcl}(F, E) \subseteq \text{hgprcl}(G, E)$.

4 HYPERSOFT GENERALIZED PRE REGULAR OPEN SETS

In this section we introduce hypersoft generalized pre regular open sets in hypersoft topological space.

Definition 4.1: A hypersoft set (F, E) in a hypersoft topological sapces (X, τ, E) is said to be hypersoft generalized pre regular open (hys gpr-open for short) set if $\text{hpint}(F, E) \supseteq (U, E)$ whenever $(F, E) \supseteq (U, E)$ and (U, E) is a hys regular closed set in (X, τ, E) .

Alternatively, A hypersoft set (F, E) is said to be a hypersoft generalized pre regular open (hys gpr-open for short) set if the complement $(F, A)^c$ is a hys gpr-closed set in (X, τ, E) .

The family of all hys gpr-open sets of a hypersoft topological space (X, τ, E) is denoted by $\text{hys gpr-open}(X, E)$.

Theorem 4.2: Let (X, τ, E) be a hypersoft topological space. Then for every $(F, E) \in \text{hys gpr-open}(X, E)$ and for every $(G, E) \in \text{hys set}(X, \tau, E)$, $\text{hpint}(F, E) \subseteq (G, E) \subseteq (F, E)$ implies $(G, E) \in \text{hys gpr-open}(X, E)$.

Proof: Let (F, E) be any hys gpr-open set of (X, τ, E) and (G, E) be any hys set of (X, E) . By hypothesis $\text{hpint}(F, E) \subseteq (G, E) \subseteq (F, E)$. Then $(F, E)^c$ is an hys gpr-closed set in (X, τ, E) and $(F, E)^c \subseteq (G, E)^c \subseteq \text{hpcl}((F, E)^c)$. By **Theorem 3.17.**, $(G, E)^c$ is a hys gpr-closed

set in (X, τ, E) . Therefore, (G, E) is a hys gpr-open set in (X, τ, E) . Hence $(G, E) \in \text{hys gpr-open}(X, E)$.

Theorem 4.3: A hypersoft set (F, E) of a hypersoft topological space (X, τ, E) is a hys gpr-open set in (X, τ, E) if and only if $(G, E) \subseteq \text{hpint}(F, E)$ whenever (G, E) is a hypersoft regular closed set in (X, τ, E) and $(G, E) \subseteq (F, E)$.

Proof: Necessity: Suppose (F, E) is a hys gpr-open set in (X, τ, E) . Let (G, E) be a hypersoft regular closed set in (X, τ, E) such that $(G, E) \subseteq (F, E)$. Then $(G, E)^c$ is a hypersoft regular open set and $(F, E)^c \subseteq (G, E)^c$. By hypothesis, $(F, E)^c$ is a hys gpr-closed set in (X, τ, E) , we have $\text{hpcl}((F, E)^c) \subseteq (G, E)^c$. Therefore $(G, E) \subseteq \text{hpint}(F, E)$.

Sufficiency: Let (U, E) be a hypersoft regular open set in (X, τ, E) such that $(F, E)^c \subseteq (U, E)$. By hypothesis, $(U, E)^c \subseteq \text{hpint}(F, E)$. Therefore $\text{hpcl}((F, E)^c) \subseteq (U, E)$ and $(F, E)^c$ is a hys gpr-closed set in (X, τ, E) . Hence (F, E) is a hys gpr-open set in (X, τ, E) .

Theorem 4.4: Let (X, τ, E) be a hypersoft topological space and hys p-open set (X, E) (resp. hys gpr-open set (X, E)) be the family of all hys p-open sets (resp. hys gpr-open sets) of (X, τ, E) . Then $\text{hys p-open set}(X, E) \subseteq \text{hys gpr-open set}(X, E)$.

Proof: Let $(F, E) \in \text{hys p-open set}(X, E)$. Then $(F, E)^c$ is hys p-closed set and so hys gpr-closed set in (X, τ, E) . This implies that (F, E) is hys gpr-open set in (X, τ, E) . Hence $(F, E) \in \text{hys gpr-open set}(X, E)$. Therefore, $\text{hys p-open set}(X, E) \subseteq \text{hys gpr-open set}(X, E)$.

5 SEPARATION AXIOMS OF HYPERSOFT GENERALIZED PRE REGULAR CLOSED SETS

In this section we have provide some applications of hypersoft generalized pre regular closed sets in hypersoft topological spaces.

Definition 5.1: If every hys gpr-closed set in (X, τ, E) is a hys p-closed set in (X, τ, E) , then the space (X, τ, E) can be called a hypersoft pre regular $T_{1/2}$ (hys pr- $T_{1/2}$ for short) space.

Theorem 5.2: An hypersoft topological space (X, τ, E) is a hys pr- $T_{1/2}$ space iff $\text{hys p-open set}(X, E) = \text{hys gpr-open set}(X, E)$.

Proof: Necessity: Let (X, τ, E) be a hys pr- $T_{1/2}$ space. Let (F, E) be a hys gpr-open set in (X, τ, E) . By hypothesis, $(F, E)^c$ is a hys gpr-closed set in (X, τ, E) and therefore (F, E) is a hys p-open set in (X, τ, E) . Hence $\text{hys p-open set}(X, E) = \text{hys gpr-open set}(X, E)$.

Sufficiency: Let $\text{hys p-open set}(X, E) = \text{hys gpr-open set}(X, E)$. Let (F, E) be a hys gpr-closed set in (X, τ, E) . Then $(F, E)^c$ is a hys gpr-open set in (X, τ, E) . By hypothesis, $(F, E)^c$ is a hys p-open set in (X, τ, E) and therefore (F, E) is a hys p-closed set in (X, τ, E) . Hence (X, τ, E) is a hys pr- $T_{1/2}$ space.

Definition 5.3: A hypersoft topological spaces (X, τ, E) is said to be a hypersoft pre regular $T_{1/2}^*$ space (hys pr- $T_{1/2}^*$ space for short) if every hys gpr-closed set is a hypersoft closed set in (X, τ, E) .

Remark 5.4: Every hys pr- $T_{1/2}^*$ space is a hys pr- $T_{1/2}$ space.

Proof: Assume (X, τ, E) is a hys pr- $T_{1/2}^*$ space. Let (F, E) be a hys gpr-closed set in (X, τ, E) . By hypothesis, (F, E) is an hypersoft closed set. Since every hypersoft closed set is a hys p-closed set, (F, E) is a hys p-closed set in (X, τ, E) . Hence (X, τ, E) is a hys pr- $T_{1/2}$ space.

Theorem 5.5: Let (X, τ, E) be a hys pr- $T_{1/2}^*$ space then,

- (i) the union of hys gpr-closed sets is hys gpr-closed set in (X, τ, E)
- (ii) the intersection of hys gpr-open sets is hys gpr-open set in (X, τ, E)

Proof: (i) Let $\{(F, E)_i\}_{i \in I}$ be a collection of hys gpr-closed sets in a hys pr- $T^*_{1/2}$ space (X, τ, E) . Thus, every hys gpr-closed sets is a hypersoft closed set. However, the union of hypersoft closed sets is a hypersoft closed set in (X, τ, E) . Therefore, the union of hys gpr-closed sets is hys gpr-closed set in (X, τ, E) .

(ii) proved by taking the complement in (i).

Acknowledgments

We studied a new class of hypersoft closed set called hypersoft generalized preregular closed sets which are weaker form of the hypersoft closed sets. Also we studied the separation axioms of hypersoft generalized preregular closed sets, namely hypersoft preregular $T_{1/2}$ space and hypersoft preregular $T^*_{1/2}$ space in hypersoft topological space. Using this idea, we are planned to study continuous function, closed functions and homeomorphisms in the hypersoft topological space.

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