Optimal Control and Analysis of Mathematical model of HIV/AIDS using Matrix Exponentiation Method

Naresh Kumar Jothi and Vadivelu .V
[1,2]Department of Mathematics, Vel Tech Rangarajan Dr.Sagunthala
R&D Institute of Science and Technology (Deemed to be University),
Avadi, Chennai, Tamilnadu, India.

Abstract

The main theme of work should be form a stage-by--stage of model mathematically concerning the dynamics and transference of HIV/AIDS. The formation of model is based on a linear system; ODE's and concedes both globally as well as locally asymptotically stable equilibria. Essentially, the boundedness and continuous of solutions are derived by using Lipchitz condition. Overseeing equations of the model are obtained systematically utilizing the matrix exponentiation via generalized eigenvectors and the principal matrix technique tends to eigenvalues. The acquired arrangement fills in as the platform for concentrating on the variables and increase of HIV/AIDS overall. Regardless, in India specific information of the illness as a help to the good results the reactions are similar finished with outcomes of model being differentiated.

Keywords: Mathematical Modelling, HIV/AIDS, Lipchitz condition, Matrix Exponentiation, Stability, ART, Equilibrium.

1. Introduction

We much of the times see people who are burdened with different diseases, many of diseases are curable. However, some diseases are not easy to cure; but extending life time using some medical treatment. AIDS [Acquired Immune Deficiency Syndrome] is one of disease not curable. Around 38.4 mm (million) Individuals living with HIV, 1.5mm (million) individuals gaining HIV and 650000 people kicked the bucket [died] because of AIDS in 2021[12,16] what's more, the sickness (disease)is at this point spreading at a high reach. It influences individuals, all things considered, variety, age, doctrine and orientation. UNAIDS (United Nations AIDS), UNICEF (United Nations International Children's Education Fund) and WHO (World Health Organization) have adapted to the situation. Huge amounts of cash are being given, and endeavors are being made to prevent the diseases from spreading. Regardless of these endeavors, the illness spreads because of an absence of information about it, its infection (HIV), the methods of transmission of the infection and a thoughtless way of life [12,16]. AIDS is brought about by the HIV infection. Variables have this name, for example, its capacity to taint just individuals, the way that it occur a shortcoming in the safe framework, and the way that infection replicates by seizing the hardware of the mankind cell. HIV is an infection that debilitates the mankind resistant framework, ultimately prompting AIDS. Also it's a final phase of contamination with infection of HIV. The improvement of AIDS could require all the greater than 7-10 years post the illness.

HIV viruses have extended begins a couple of occasions in the US to from one side of the planet to the other in only 22 years. Updated July 2022, it was enumerated that 36.7 mm (million) young's and 1.7 mm (million) kids younger than 16 around the globe were living with HIV/AIDS [9, 12]. As of now, in the country of South Africa is the globally largest number individuals infected by HIV, trailed by India and Nigeria separately. In our country India the main HIV virus affected patient was recognized in 1986 and first AIDS patient in Chennai was recognized in 1986. In India Official appraisals of individual run their life with HIV/AIDS from range of 2.5-3.5 mm (million) starting around 2021[9, 12].

As an issue of treat, it is basic to concentrate on the dynamics and transference of the illness to limit it's growing of the prosperity of mankind existence. In this respect, modelling of mathematical assumes a fundamental part and supports in comprehending the transference conduct and spread of such illnesses between various populace gatherings, through creating followed by experimental examinations in mathematical models [5]. Among various specialists have focused on the approach to acting, growing and transference of the HIV/AIDS in different populace classes through numerical demonstrating, to help scientists and the associated clinical investigators to diminish and restrict the increase of the ailment. Mathematically examined the increase of AIDS epidemic with development of HIV effect by Naresh et al., [14] in a populace of fluctuating size. Junjie et al., [17] focused on the unique HIV/AIDS model of mathematical transference and jurisdiction.

Propelled by the previously mentioned jobs, in India our point is to form a model of mathematical to work and foresee the way of behaving, dynamics and extent of HIV/AIDS, by taking into account the various scales of control. In this representation is planned utilizing the compartmental investigation and depends on a structure of four straight linear ODE's. The suggested job should help biologists and researchers in the area of medical to limit of the epidemic layout.

2. Mathematical Model Conceptualization

On this segment, from India we will officially foster an original epidemic model to concentrate on the HIV/AIDS of dynamics and transference and the model is reproduce utilizing the information of HIV/ AIDS. The conceptualization of the model depends on a few designs of classical, for e.g., SI, SIR and SIS models.

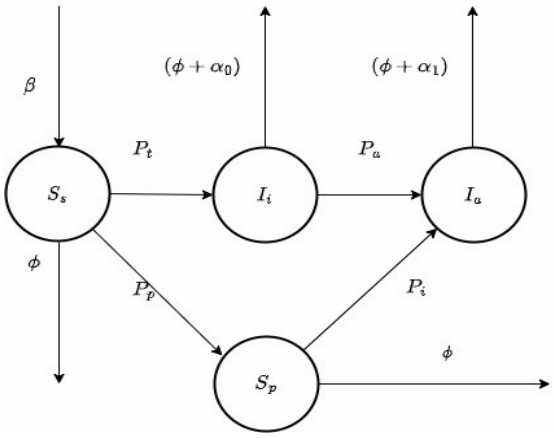

Diagram 1. Diagrammatic portrayal in the formed HIV/AIDS model. Progress of populace shown stage-by-stage through the arrows.

2(a). Classification and Notations. From this boundaries and notations are utilized in the model of conceptualized;

β= The recruitment rate

This mathematical model is represented is analytically by a stage-by-stage of diagrammed as mentioned in diagram (1). The boundaries act for the transference-rates between the communicating stages populace divisions, shown in diagram (1).

2(b). Presumptions.

Following presumptions are built for improve on the mathematical model for researching the dynamics and transference of HIV/AIDS (see [8], [9]):

(i) The consequent populace of the suggested system whenever is the count of populaces of various divisions

(ii) Continuously non-negative function of time t for each stage of populace.

(iii) Because of HIV/AIDS death happen just in the stages.

(iv) Transference of all the ranges in between specific populace stages and the passing ranges are steady for a specific period of time, such a lot of that all ranges are non-negative.

(v) HIV/AIDS recuperated stage is invulnerable.

(vi) At any time of moment total populace stays saved.

2(c). Overseeing Equations.

The conceptualized HIV/AIDS mathematical model appeared visually in diagram(1) depends on Mathematical modelling and represented by the four system of linear ODE's [4] which are shown up at, involving the preservation regulations as the people either pass on because of HIV-AIDS /other explanation or are just moved to a various stage. Because of this explanation, these equations are shown below.

(2.1)

(t)

(t)

As given initial circumstances;

(2.2)

Reasonably the initial circumstances are picked as in each populace class; the quantity of people is more noteworthy or equivalent to nothing. Anyway the sickness to shake out in the populace there should be a few infectious people and at first entire of the population with the exception of the Infective class is thought of as powerless to the illness.

The framework (2.1) addresses a period subordinate model that portrays the quantity of people groups in each division,,(t) and (t) at various periods. Obviously the infection advances inside the populace, the shake out of the HIV/AIDS dynamically explainable utilizing these equations.

3. Model Investigation and Possessions

From this segment, the analysis of model and its subjective possessions, comprising the invariance of region, the results of uniqueness and existence, locally as well as globally asymptotically stability and their equilibria, are explained.

3(a) Invariance of Region: Utilizing presumptions (i) and (vi), we get;

(3.1)

Subsequently we have

(3.2)

Presently, utilizing Eq. (2.1) in Eq. (3.2), we get that;

Afterward the and stages are expected that passing because of HIV/AIDS happen, subsequently in class and the demise range because of AIDS are utilizing in above equation, we obtain;

(3.3)

Improving in equation (3.3), we obtain,

This is related to integrating factor of linear I.F.

The G.S of (3.3) is acquired as;

Identically,

(3.4)

Using we have;

(3.5)

From equation (3.4) and (3.5) we have;

Thus, the G. of inequity (3.4) can be communicated as;

(3.6)

Where and. From inequity (3.6), that's what it follows,

Hence, the region is given by;

$\mathbb{R}=$ act for the region feasible for the model and stays limited, where the all out populace is non-negative.

3(b) Solution of the Uniqueness and Existence

To concentrate on transference of dynamics of HIV/AIDS between different populace stages utilizing the suggested representation, demonstrating the solution of uniqueness and existence of the framework is basic. We have the accompanying outcomes.

[1, 15] A function satisfies the Lipchitz condition on a rectangle with respect to the variables, if there exist a such that

Where is Lipchitz constant. Then a non negative constant satisfyingthere is a solution of vector is unique.

All the partial derivatives are bounded and continuous in, if is relevant to make reference to that the prerequisite for the result to hold well. From the initial circumstances (2.2) and

the system of equations (2.1), uniquely exists in $\forall\ t \geq 0$ in the representation of the initial value problem of solution.

Assume that;

Besides, taking, it obviously follows that are bounded and continuous. Therefore, the system of equations (2.1) ensures that its outcome solution has unique.

3(c) Stability Analysis and essence of Equilibria.

From the system of equations (2.1) we simultaneously solving the system, by equating zero to the right side of the system and we get the equilibrium points. Finally two equilibrium points are founded. Namely one is free of disease equilibrium and another one is the equilibrium of epidemicwhere;

with and in this case,

and,

Such that;

, ,

For the testing purpose of the equilibrium points, arrange the system of equations (2.1) in the form of Jacobian matrix.

J (3.7)

The eigen value of the equation is:

Where.

Hence its eigenvalues which are negative (at all equilibrium points). Thusly, both are locally as well as globally asymptotically stable form all equilibria [3, 15].

4. Result of the Model

We have formed the matrix from the system of equations (2.1)

(4.8)

Where denotes the transpose of the matrix and.

Assume that the fundamental matrix denotes (see [3], [13], [15]) relating to the homogeneous part of equation (2.1), then the solution uniqueness [5, 6] of the system is obtained as

(4.9)

5. Simulations of Numerical values

From this segment, we carry out a little count of simulations of numerical values to portray the dynamics and transference of HIV/AIDS by utilizing the model proposed and the result of Equation (4.9) obtained. In the Table (1) the simulator processes are given by using the numerical datum of the various parameters.

DATA TABLE 1. Numerical/reasonable (average) values are tabled and engaged with the formulated model upsides of different parameters per day in India.

Parameter	Value	Parameter	Value
	380000		325800
	325000		12000
	1100		319400
	7000		6400

In Table (1) values are used to found the eigenvalues of the Jacobian matrix J as

Appearing in Equation (4.9), the Fundamental matrix H (t) is given by;

(5.10)

Where., is the corresponding eigenvector to the eigenvalues of J for the matrix of Jacobian.

(5.11)

The eigenvectors, are obtained, by using Matrix Exponentiation via generalized eigenvectors and afterward to obtain the solution of uniqueness utilized in equation (4.9), with in the populace portraying the dynamics and transference of HIV/AIDS.

Conclusion

From the current review, based on a linear- system of ODE's, we formed a model of mathematical stage-by-stage. In India, with respect to the dynamics and transference of the irresistible illness (HIV/AIDS). Total of all populace is separated into various populace divisions. Equations shown that the model concedes solutions of unique and same are obtained systematically utilizing the matrix exponentiation via generalized eigenvectors and the principal matrix technique tends to eigenvalues and concedes both globally as well as locally asymptotically stable equilibria. Essentially, the boundedness and continuous of solutions are derived exhaustively.

Moreover, prediction of the figured out model that there has not been any reasonable diminishing in the powerful cases, as till date, which is very reasonable and underwrites the authenticity of the model proposed that there is not been an accessibility of a suitable medication to vanish the HIV virus. In globe as well as in India, as such that, the outcomes may be the basic of proposed model for focusing on the dynamics and transference of HIV/AIDS. Additionally, the researchers and biologists of various nations used this model to assess their strategies and status of epidemic infection to diminish and control. Moreover, We make mindfulness viewing individual security as well as control the spread of infection.

Author's Contributions: The authors have equal contributions. All authors drafted the paper, read and approved final version of this paper.
Conflicts of Interest: The author declares no conflict of interest.
Data Availability: All of the necessary data required for this article is included within this article.
Funding Information: No funding is available regarding this article.

REFERENCES

[1] An Introduction to ordinary Differential Equations. Earl A. Coddington, Dover Publications, INC, Newyork.1989.

[2] D. Huang, et al., Analysis of an HIV infection model with treatments and delayed immune response, Applied Mathematical Modelling, 40, No. 4 (2016) 3081-3089.

[3] Fundamentals of Matrix Analysis with Applications, Edward Barry Saff, Arthur David Snider, Wiley.2016.

[4] F.Brauer, et al., Compartmental Models in Epidemiology, Mathematical Epidemiology, 1945, (2008) 1979.

[5] F. A. Zargar and M.A. Khanday, Mathematical analysis on the dynamics of COVID-19 in India using SIR Epidemic Model, Mapana Journal of Sciences, 19, No. 3 (2020) 19.

[6] H. Huo, R. Chen and X. Wang, Modelling and stability of HIV/AIDS epidemic model with treatment, Applied Mathematical Modelling, 40, (2016) 6550-6559.

[7] Indian HIV-AIDS Estimates 2021 Report, National AIDS Control Organisation ICMR National Institute of Medical Statistics Ministry of Health & Family Welfare, Govt. of India.

[8] MA. Khanday, Introduction to Modeling and Biomathematics, Dilpreet Publishing House, New Delhi. 2016.

[9] Ministry of Health & Family Welfare, Govt. of India, www.mohfw.gov.in

[10] Naresh Kumar Jothi, Suresh, M.L., Malini, T N M., Mathematical Model for the control of life cycle of Feminine Anopheles Mosquitoes International Journal of Recent Technology and Engineering (IJRTE) ISSN: 2277-3878, Volume-8 Issue-3 (2019)

[11] Naresh Kumar Jothi and Suresh Rasappan, Stabilization and Complexities of Anopheles Mosquito Dynamics with Stochastic Perturbations, IAENG International Journal of Applied Mathematics, 47(3), 307-311(2017).

[12] National AIDS Control Organisation, http://naco.gov.in/hiv-facts-figures

[13] P. Hartman, Ordinary Differential Equations, Classics in Applied Mathematics, 38 No. 2 (2010).

[14] R. Naresh, A. Tripathi and D. Sharma, Modelling and analysis of the spread of AIDS epidemic with immigration of HIV infectives, Mathematical and Computer Modelling, 49, (2009) 880-892.

[15] Shepley L. Ross, Differential Equations, WILEY Third Edition, 2014.

[16] World Health Organisation, www.who.int

[17] W. Jun-jie , et al., Dynamic mathematical models of HIV/AIDS transmission in China, Chin Med J (Engl), 123, No. 15 (2010), 2120-2127.

[18] Z. Mukandavire, et al., Mathematical analysis of a sex-structured HIV/AIDS model with a discrete time delay, Nonlinear Analysis, 71, (2009) 1082-1093.