Rayleigh Wave Propagation in a Rotating Orthotropic Elastic Solid Half-space under Gravity and Initial Stress with Impedance Boundary Conditions

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Abstract: The effects of initial stress and angular rotations are investigated on Rayleigh type surface wave propagation in an orthotropic elastic solid in its gravity field. Employing the method of plane harmonic wave solution, the governing equations are solved to derive the dispersion relation of Rayleigh wave propagation with using impedance boundary conditions. The effect of initial stress and rotation are shown graphically on Rayleigh wave velocity for a particular solid.

Keywords: Rayleigh surface waves, orthotropic material, initial compression, gravity and impedance boundary conditions

1 INTRODUCTION

Many authors focused their attention on the study of surface waves at half-spaces and interfaces of different elastic solids. The information obtained from the behavior of surface wave propagation is relevant to geophysicists and seismologists to locate the earthquakes. Such type of information is very essential to calculate earthquake energy and the mechanism of global tectonics.

Lord Rayleigh [1] prescribed the waves in the year 1885 are known as Rayleigh waves. These are surface type of waves and they having two typesof motion named as ‘ longitudinal motion’ and ‘ transverse motion’. Rayleigh waves are creating an elliptic motion. Rayleigh waves are destructive type of elastic wavesand travel with low velocity. The wave propagation studies in different composite media are very essential in the behavior of earth’s interior. The ‘stress’ that occursin a body under the absence of external body forces is called “initial stress”. Theinitial stress in the media caused by physical considerations like cold working, the gradual creep process and gravity changes. Also the surface of earth is an elastic medium that is subjected to significant initial stresses. Many science and technology branches extensively use surface wave problems in an orthotropic elastic solid. In the second half of 20^{th} century, many investigations in surface wave propagations had been started on development of polymer science, plastic industry, geology and engineering sciences. Theoretical study with applications on surface wave propagation has become very important task for mechanics of solid.

By assuming traction free boundary conditions i.e., stress vanished surfaces, the Rayleigh type surface wave problems are solved. In geophysics or seismology problems, another type of boundary conditions is considered. Electromagnetism and acoustic problems are commonly solved by using impedance boundary conditions. The linear combination of unknown function and their derivative are the impedance boundary conditions.

Mandeep Singh et al. [2] studied the Rayleigh wave propagation problem on orthotropic elastic solids with two temperatures in context of thermo elasticity. The effect of gravity on Rayleigh waves was investigated by Biot [3]. Nahed et al. [4] studied the effect of rotation on non-homogeneous infinite cylinder of orthotropic material. The effect of gravity on surface and Rayleigh type surface waves are studied by many authors Ali Mubaraki et al. [5], Aftab Khan et al. [6], Rajneesh Kakar , Shikha

Kakar , Kanwaljeet Kaur and Kishan Chand Gupta [7]. Narottam Maity et al. [8] investigated the effect of gravity and surface stress on Rayleigh wave propagation in Fiber-Reinforced Half Space. In very recent, rotation effect on Rayleigh type waves in a micropolar elastic medium with stretch in its gravity field was studied by Somaiah [9].

Tiersten [10] discussed the impedance boundary conditions in wave propagation studies. The secular equations of Rayleigh waves are derived by Malischewsky [11] by modifying Tiersten's conditions in terms of displacements and stresses. Duran, Godoy and Nedelac [12], Vinh and Hue [13] and Singh [14] derived the secular equations of Rayleigh waves under impedance boundary conditions.

Many researchers discussed separately the effects of angular rotation and initial stress on Rayleigh surface waves with existing boundary conditions. But in this article, we discussed the effects of rotation and initial stress on Rayleigh wave propagation in orthotropic elastic half-space in its gravity field with impedance boundary conditions. The results are presented graphically for a particular numerical example with the help of derived secular equations.

2 PROBLEM FORMATION

Let origin of the coordinate system at any point on the plane surface be $oxyz$, the initial compression P be along x-axis and y-axis be pointing vertically downward in to the half space. With this, it is represented by $y \geq 0$ and stress free surface is y=0 and assume that the wave is propagating along x-axis. With this, all the particles on the line parallel to z-axis are equally displaced and the field equations are not depending on the z-coordinate. Choose the medium is rotating about z-axis with the speed of angular rotation Ω. So, angular rotation vector $\vec{\Omega}$ taken as $\vec{\Omega} = (0,0,\Omega)$ with two types of accelerations named as Coriolis acceleration $2\left(\vec{\Omega} \times \vec{u}\right)$ and centripetal acceleration $\vec{\Omega} \times \left(\vec{\Omega} \times \vec{u}\right)$. The wave is two dimensional in xy-plane, so, macro displacement vector

$\vec{u} = (u, v, w)$ with $u = u(x, y, t)$, $v = v(x, y, t)$ and $w = 0$. Let the acceleration due to gravity be *g*. Body force components are $(X, Y) = (0, -g)$. Choose that the initial stress due to gravity is hydrostatic. Datta [16] introduced stress tensor σ_{ij} as follow

$$\sigma_{xx} = \sigma_{yy} = \sigma \quad , \quad \sigma_{xy} = 0 \tag{1}$$

The equilibrium relations for the initial stress field also given by Datta [16] as

$$\frac{\partial \sigma}{\partial x} = 0 \; , \quad \frac{\partial \sigma}{\partial y} = \rho g \tag{2}$$

where ρ is the mass density of the material.

With the use of equations (1) and (2), the equations of motion of an orthotropic elastic solid involving initial stress *P* and gravity *g* in three dimensional form is given by [Ref. [3], chapters 3 and 5]

$$\rho[\ddot{u} - \Omega^2 u + 2\Omega\dot{v}] = \sigma_{xx,x} + \sigma_{xy,y} + \sigma_{xz,z} + P[\phi_{xy,y} - \phi_{xz,z}] - \rho g v_{,x} \tag{3}$$

$$\rho[\ddot{v} - \Omega^2 v - 2\Omega\dot{u}] = \sigma_{xy,x} + \sigma_{yy,y} + \sigma_{yz,z} - P\phi_{xy,x} + \rho g u_{,y} \tag{4}$$

$$\rho\ddot{w} = \sigma_{zx,x} + \sigma_{zy,y} + \sigma_{zz,z} - P\phi_{xz,x} \tag{5}$$

where ϕ_x, ϕ_y, ϕ_z are micro-rotational vector components along *x, y, z* directions respectively. The superpose dot indicates partial derivative with respect to time variable *t*, partial derivative with respect to the coordinate axis is denoted by the comma followed and

$$\phi_{ij} = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) \tag{6}$$

The equations (3) to (5) in two-dimensional xy-plane reduces to

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}+P\frac{\partial \phi_{xy}}{\partial y}-\rho g\frac{\partial v}{\partial x}=\rho\left[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial v}{\partial t}\right] \tag{7}$$

$$\frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}-P\frac{\partial \phi_{xy}}{\partial x}+\rho g\frac{\partial u}{\partial y}=\rho\left[\frac{\partial^2 v}{\partial t^2}-\Omega^2 v-2\Omega\frac{\partial u}{\partial t}\right] \tag{8}$$

and the stress σ_{ij} can be written as

$$\sigma_{xx}=(c_{11}+P)\frac{\partial u}{\partial x}+(c_{12}+P)\frac{\partial v}{\partial y} \tag{9}$$

$$\sigma_{yy}=c_{12}\frac{\partial u}{\partial x}+c_{22}\frac{\partial v}{\partial y} \tag{10}$$

$$\sigma_{xy}=c_{44}\left(u_{,y}+v_{,x}\right) \tag{11}$$

Where c_{ij} ; i, j =1, 2, 3, 4 are the stiffness tensor components in the contraction notation and the problem is in two dimensional xy-plane, so that $c_{13}=c_{33}=c_{32}=0$.

Inserting equations (9) to (11) in equations (7) and (8) also considering above assumptions with $c_{44}=\frac{1}{2}(c_{11}-c_{12})$,we get equations (7) and (8) in terms of displacement components *u, v, w* as

$$(c_{11}+P)\left[2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right]+c_{12}\left[\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}\right]-2\rho g\frac{\partial v}{\partial x}=2\rho\left[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial v}{\partial t}\right] \tag{12}$$

$$c_{11}\left[\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right]+(c_{12}-P)\left[\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial^2 v}{\partial x^2}\right]+2c_{22}\frac{\partial^2 v}{\partial y^2}+2\rho g\frac{\partial u}{\partial x}=2\rho\left[\frac{\partial^2 v}{\partial t^2}-\Omega^2 v-2\Omega\frac{\partial u}{\partial t}\right] \tag{13}$$

The derivable displacement components $u(x, y, z)$, $v(x, y, z)$ can be expressed in terms of Helmholtz functions $H(x, y, t), M(x, y, t)$, $\left(i.e., (u, v, 0)=\nabla H+\nabla\times M\right)$ as

$$u=\frac{\partial H}{\partial x}+\frac{\partial M}{\partial y}\quad,\quad v=\frac{\partial H}{\partial y}-\frac{\partial M}{\partial x} \tag{14}$$

After inserting eq. (14) in eqns. (12) and (13) we obtain

$$(c_{11}+P)\nabla^2 H+\rho g\frac{\partial M}{\partial x}=\rho\left[\frac{\partial^2 H}{\partial t^2}-\Omega^2 H+2\Omega\frac{\partial H}{\partial t}\right] \tag{15}$$

$$(c_{11}+P)\nabla^2 M-2c_{12}\frac{\partial^2 M}{\partial x^2}-2\rho g\frac{\partial H}{\partial x}=2\rho g\left[\frac{\partial^2 M}{\partial t^2}-\Omega^2 M+2\Omega\frac{\partial M}{\partial t}\right] \tag{16}$$

and

$$c_{11}\left(\frac{\partial^2 M}{\partial y^2}-\frac{\partial^2 M}{\partial x^2}\right)+(c_{12}-P)\nabla^2 M-2c_{22}\frac{\partial^2 M}{\partial y^2}+2\rho g\frac{\partial H}{\partial x}=2\rho\left[\Omega^2 M-\frac{\partial^2 M}{\partial t^2}-2\Omega\frac{\partial H}{\partial t}\right] \tag{17}$$

$$c_{11}\frac{\partial^2 H}{\partial x^2}+c_{22}\frac{\partial^2 H}{\partial y^2}+\rho g\frac{\partial M}{\partial x}=\rho\left[\frac{\partial^2 H}{\partial t^2}-\Omega^2 H-2\Omega\frac{\partial M}{\partial t}\right] \tag{18}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

3 SOLUTION OF THE PROBLEM

Due to the initial compressive, wave taken in the *x*-direction only, the body wave velocities are different in the *x* and *y* directions. The compressive waves along *x* and *y* directions are given in equations (15) and (18) respectively and shear waves along *x* and *y* directions are respectively given in equations (17) and (16). Our concentration is Rayleigh wave propagation in *x*-direction only. So, we consider the equations (15) and (17).

For harmonic wave propagation along horizontal direction (i.e., along *x*-axis), we may assume the solutions of equations (15) and (17) in the following form of

$$H=F(y)exp[ik(x-vt)] \tag{19}$$

$$M=G(y)exp\left[ik\left(x-vt\right)\right] \tag{20}$$

where F and G are amplitude ratios, k and v are respectively the wave number and wave velocity connected with the angular frequency ω with $\omega = \dfrac{v}{k}$,

Substituting equations (19) and (20) in equations (15) and (17), we obtain the following differential equations

$$(D^2+\alpha^2)F(y)+\beta^2 G(y)=0 \tag{21}$$

and

$$(D^2+\gamma^2)G(y)+\delta^2 F(y)=0 \tag{22}$$

where

$$D^2=\frac{\partial^2}{\partial y^2}; \qquad \alpha^2=\frac{\rho\left(v^2k^2+2i\Omega vk+\Omega^2\right)}{(c_{11}+P)}-k^2; \qquad \beta^2=\frac{i\rho gk}{(c_{11}+P)};$$

$$\gamma^2=\frac{(c_{11}+P-c_{12})k^2-2\rho\left(\Omega^2-v^2k^2\right)}{\left(c_{11}+c_{12}-P-2c_{22}\right)}; \qquad \delta^2=\frac{2\rho\left(ikg-2i\Omega kv\right)}{(c_{11}+c_{12}-P-2c_{22})} \tag{23}$$

On using eq. (21) in eq. (22) we get the following fourth order differential equation for F(y)

$$(D^4+aD^2+b)F(y)=0 \tag{24}$$

where

$$a=\alpha^2+\gamma^2; \quad b=\alpha^2\gamma^2-\beta^2\delta^2 \tag{25}$$

Equation (24) can be written in the form of

$$(D^2+m_1^2)(D^2+m_2^2)F(y)=0 \tag{26}$$

With

$$m_1^2+m_2^2=a=\alpha^2+\gamma^2; \quad m_1^2m_2^2=b=\alpha^2\gamma^2-\beta^2\delta^2$$

$$m_1^2; m_2^2 = \frac{1}{2}\left[\left(\alpha^2+\gamma^2\right) \pm \sqrt{\left(\alpha^2-\gamma^2\right)^2+4\beta^2\delta^2}\right] \tag{27}$$

Since the solution of eq. (26) must be exponential nature, so assume its solution of the form

$$F(y) = A exp(-im_1 y) + B exp(im_1 y) + A^* exp(-im_2 y) + B^* exp(im_2 y) \tag{28}$$

where A, B, A and B are constants.

For Rayleigh type waves, the displacement must be zero as $y \to \infty$. So the constants corresponding to B, B must vanish and m_1, m_2 are taken to be imaginary. So, eq. (28) reduces to

$$F(y) = A exp(-im_1 y) + A^* exp(-im_2 y) \tag{29}$$

So, that eq. (19) reduces to

$$H(y) = \left[A exp(-im_1 y) + A^* exp(-im_2 y)\right] \exp\left[ik(x-vt)\right] \tag{30}$$

Inserting eq. (29) in eq. (21) we get the function $G(y)$ as

$$G(y) = A\zeta_1 exp(-im_1 y) + A^*\zeta_2 exp(-im_2 y) \tag{31}$$

Where

$$\zeta_j = -\frac{(m_j^2+\alpha^2)}{\beta^2} \; : j = 1,2 \tag{32}$$

From eq. (20) the potential function M becomes

$$M = \left[A\zeta_1 exp(-im_1 y) + A^*\zeta_2 exp(-im_2 y)\right] \exp\left[ik(x-vt)\right] \tag{33}$$

On using equations (30) and (31) in eq. (14), the macro displacement components $u(x, y, t)$, $v(x, y, t)$ becomes

$$u(x,y,t)=\left[i(k-m_1\zeta_1)A exp(-im_1y)+i(k-m_2\zeta_2)A^* exp(-im_2y)\right]\exp\left[ik(x-vt)\right] \quad (34)$$

and

$$v(x,y,t)=\left[-i(m_1+k\zeta_1)A exp(-im_1y)-i(m_2+k\zeta_2)A^* exp(-im_2y)\right] exp\left[ik(x-vt)\right] \quad (35)$$

4 BOUNDARY CONDITIONS AND SECULAR EQUATION

The general form of two dimensional impedance boundary conditions in terms of displacements and stresses are presented by Malischewsky [11] as follows:

$\sigma_{j2}+\Sigma_j u_j=0$ for $y=0$, where Σ_j represents the impedance parameters with the dimensions of stress / length. For elastic half-space, Godoy, Duran and Nedelec [12] expressed Σ_j as $\Sigma_j=\omega Z_j$ where Z_j are impedance real parameters with dimensions are stress / velocity and circular frequency ω is defined by $\omega=kv$. So, the impedance boundary conditions at the surface $y=0$ of an orthotropic elastic medium are

$$\sigma_{2j}+\omega Z_j u_j=0,\ j=1,2 \text{ or } j=x,y \text{ or } \sigma_{yj}+\omega Z_j u_j=0$$

which can be written as

$$\sigma_{yx}+\omega Z_1 u=0;\quad \sigma_{yy}+\omega Z_2 v=0 \quad (36)$$

With the help of eq. (10), (11), (34) and (35), the boundary conditions (36) on the surface $y=0$ reduces to the following system of two homogeneous equations

$$\begin{gathered}\left[c_{44}k(m_1+k\zeta_1)+(k-m_1\zeta_1)(c_{44}m_1+i\omega Z_1)\right]A+\left[c_{44}k(m_2+k\zeta_2)+(k-m_2\zeta_2)(c_{44}m_2+i\omega Z_1)\right]A^*=0\\ \left[c_{12}k(k-m_1\zeta_1)+(m_1+k\zeta_1)(c_{12}m_1+i\omega Z_2)\right]A+\left[c_{12}k(k-m_2\zeta_2)+(m_2+k\zeta_2)(c_{12}m_2+i\omega Z_2)\right]A^*=0\end{gathered} \quad (37)$$

Eliminating A, A^* in the system (37), one can obtain the following secular equation for Raleigh waves in an orthotropic elastic solid which is the function of initial stress, gravity and impedance boundary parameters.

$$\frac{c_{44}k(m_1+k\zeta_1)+(k-m_1\zeta_1)(c_{44}m_1+i\omega Z_1)}{c_{44}k(m_2+k\zeta_2)+(k-m_2\zeta_2)(c_{44}m_2+i\omega Z_1)}=\frac{c_{12}k(k-m_1\zeta_1)+(m_1+k\zeta_1)(c_{12}m_1+i\omega Z_2)}{c_{12}k(k-m_2\zeta_2)+(m_2+k\zeta_2)(c_{12}m_2+i\omega Z_2)} \quad (38)$$

5 PARTICULAR CASES

(i) When the angular rotation is neglected (i.e., $\Omega = 0$), then the equation (38) reduces to

$$\frac{c_{44}k\left(m_1^* + k\zeta_1^*\right) + (k - m_1^*\zeta_1^*)\left(c_{44}m_1^* + i\omega Z_1\right)}{c_{44}k\left(m_2^* + k\zeta_2^*\right) + (k - m_2^*\zeta_2^*)\left(c_{44}m_2^* + i\omega Z_1\right)} = \frac{c_{12}k(k - m_1^*\zeta_1^*) + \left(m_1^* + k\zeta_1^*\right)\left(c_{12}m_1^* + i\omega Z_2\right)}{c_{12}k(k - m_2^*\zeta_2^*) + \left(m_2^* + k\zeta_2^*\right)\left(c_{12}m_2^* + i\omega Z_2\right)} \quad (39)$$

where

$$m_1^{*2} : m_2^{*2} = \frac{1}{2}\left[(\alpha^{*^2} + \gamma^{*^2}) \pm \sqrt{(\alpha^{*2} - \gamma^{*2})^2 + 4\beta^2\delta^{*^2}}\right] ;$$

$$\zeta_j^* = \frac{(m_j^{*2} + \alpha^{*2}}{\beta^2}; j = 1, 2; \qquad \alpha^{*2} = \frac{\rho\omega^2}{c_{11} + P} - k^2; \quad (40)$$

$$\gamma^{*^2} = \frac{k^2(c_{11} + P - c_{12}) + 2\rho\omega^2}{c_{11} + c_{12} - P - 2c_{22}}; \qquad \delta^{*^2} = \frac{2\rho ikg}{c_{11} + c_{12} - P - 2c_{22}}$$

Equations (39) represents the frequency equation of Rayleigh waves in a non- rotating orthotropic elastic medium with initial stress, impedance boundary parameters in its gravity field.

(ii) After neglecting initial stress (i.e., $P = 0$), one can reduce eq. (38) as

$$\frac{c_{44}k(m_1' + k\zeta_1') + (k - m_1'\zeta_1')(c_{44}m_1' + i\omega Z_1)}{c_{44}k(m_2' + k\zeta_2') + (k - m_2'\zeta_2')(c_{44}m_2' + i\omega Z_1)} = \frac{c_{12}k(k - m_1'\zeta_1') + (m_1' + k\zeta_1')(c_{12}m_1' + i\omega Z_2)}{c_{12}k(k - m_2'\zeta_2') + (m_2' + k\zeta_2')(c_{12}m_2' + i\omega Z_2)} \quad (41)$$

where

$$m_1'^2 m_2'^2 = \frac{1}{2}\left[\left(\alpha'^2 + \gamma'^2\right) \pm \sqrt{(\alpha'^2 - \gamma'^2)^2 + 4\beta'^2\delta'^2}\right]$$

$$\zeta_j' = \frac{(m_j'^2 + \alpha'^2)}{\beta'^2}; j = 1, 2; \qquad \alpha'^2 = \frac{\rho(\omega + 2i\Omega\omega + \Omega^2)}{c_{11}} - k^2 \quad (42)$$

$$\beta'^2 = \frac{i\rho gk}{c_{11}}; \; \gamma'^2 = \frac{(c_{11} - c_{12})k^2 - 2\rho(\Omega^2 - \omega^2)}{c_{11} + c_{12} - 2c_{22}}; \; \delta'^2 = \frac{2\rho(ikg - 2i\Omega kv)}{c_{11} + c_{12} - 2c_{22}}$$

Equation (41) represents dispersion relation of Rayleigh surface waves in a rotating orthotropic elastic solid with impedance boundary parameters in its gravity field.

(iii) In the absence of rotation and initial stress (i.e., $\Omega = 0$, $P = 0$), eq. (38) reduces to

$$\frac{c_{44}k\left(n_1 + k\eta_1\right) + (k - n_1\eta_1)\left(c_{44}n_1 + i\omega Z_1\right)}{c_{44}k\left(n_2 + k\eta_2\right) + (k - \eta_2 n_2)\left(c_{44}n_2 + i\omega Z_1\right)} = \frac{c_{12}k\left(k - n_1\eta_1\right) + (n_1 + k\eta_1)(c_{12}n_1 + i\omega Z_2)}{c_{12}k\left(k - n_2\eta_2\right) + (n_2 + k\eta_2)(c_{12}n_2 + i\omega Z_2)} \quad (43)$$

Where

$$n_1^2, n_2^2 = \frac{1}{2}\left[(\alpha_0^2 + \gamma_0^2) \pm \sqrt{(\alpha_0^2 - \gamma_0^2)^2 + 4\beta_0^2\delta_0^{\ 2}}\right]; \ \eta_j = \frac{(n_j^2 + \alpha_0^2)}{\beta_0^2}; j = 1,2$$

$$\alpha_0^2 = \frac{\rho\omega^2}{c_{11}} - k^2; \ \beta_0^2 = \frac{i\rho g k}{c_{11}}; \quad \gamma_0^2 = \frac{(c_{11} - c_{12})k^2 + 2\rho\omega^2}{(c_{11} + c_{12} - 2c_{22})}; \quad \delta_0^{\ 2} = \frac{2i\rho k g}{c_{11} + c_{12} - 2c_{22}} \quad (44)$$

Equation (43) represents dispersion relation of Rayleigh surface waves in a non-rotating orthotropic elastic solid with impedance boundary parameters in its gravity field.

6 NUMERICAL EXAMPLE AND DISCUSSIONS

To discuss the theoretical results, we consider a particular numerical example as an orthotropic medium; so, consider relevant parameters from Abd-Alla et al. [15] as under:

$c_{11} = 2.694; \ c_{12} = 0.661; \ c_{22} = 2.363$; $c_{44} = \frac{1}{2}\left(c_{11} - c_{12}\right)$; Gravitational force of the earth as $g = 9.8$ m/sec^2; mass density $\rho = 6$.

At wave length l=10 cm, the natural wave number $k = 2\pi / l$ and take the natural angular frequency ω = 10 rad / sec. Initial stress effects in a rotating and non rotating solid on the Rayleigh wave velocity against the non-dimensional impedance parameter Z_1 are shown in fig.1 to fig.3.

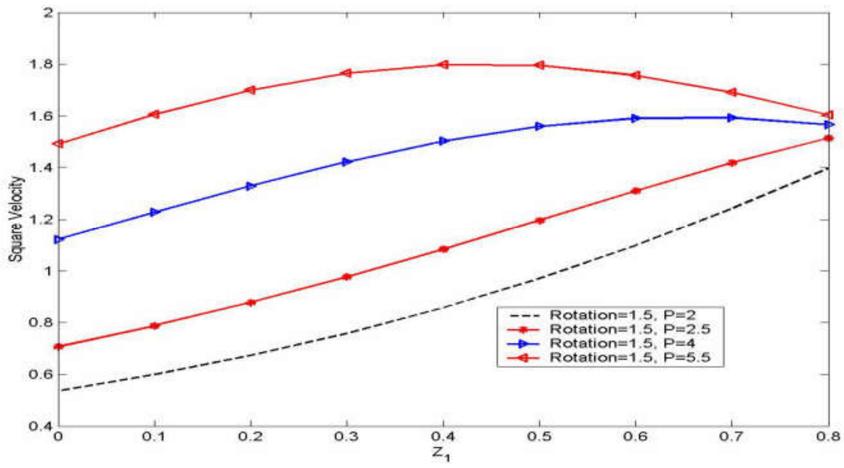


FIGURE 1: Impedance parameter Z_1 versus square velocity of Rayleigh wave for rotation =1.5.

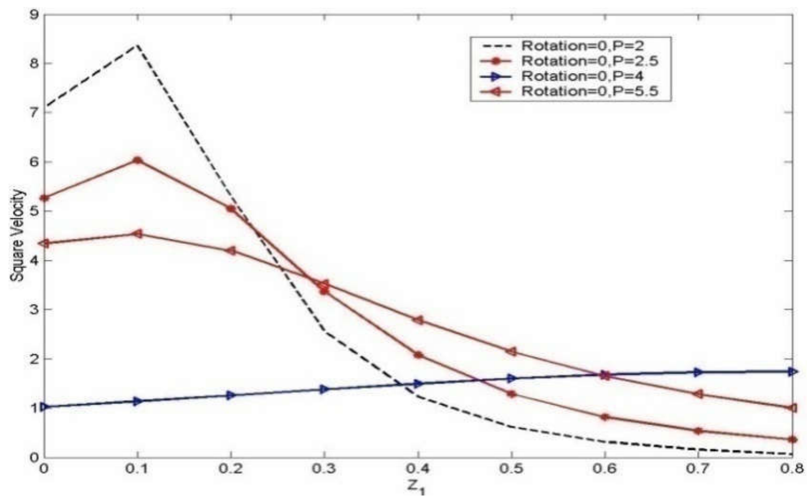


FIGURE 2: Impedance parameter Z_1 versus square velocity of Rayleigh wave for rotation=0.

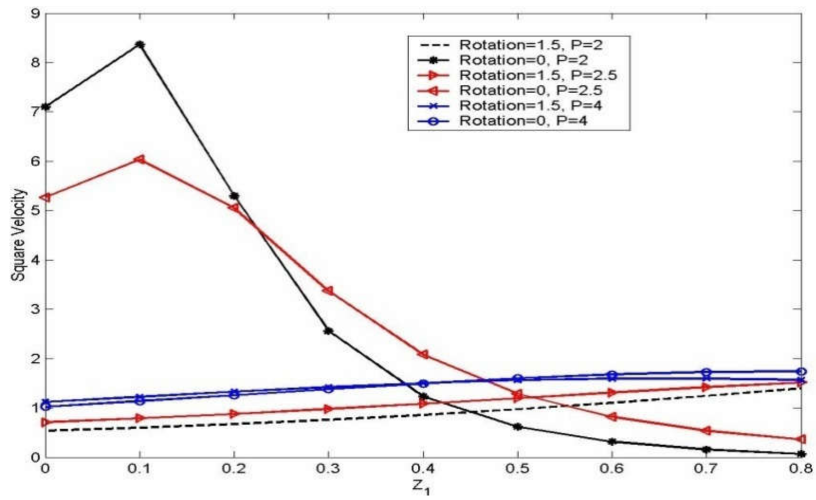
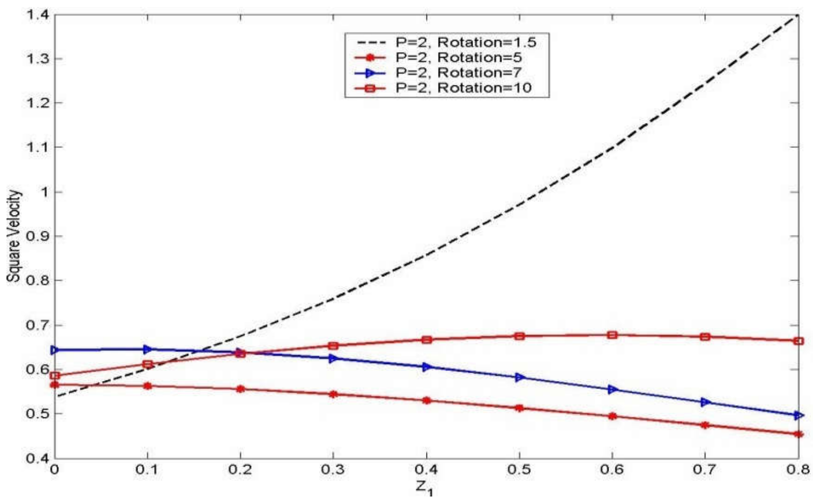


FIGURE 3: Impedance parameter Z_1 versus square velocity of Rayleigh wave for rotating, non-rotating and initial stresses.

It is observed that the Rayleigh wave velocity is increases with the increasing initial stress in the rotating solid for the range of Z_1 parameter with $0 \le Z_1 \le 0.8$.The initial stress is inverse proportional to the Rayleigh wave velocity in non-rotating solid at very low impedance parameter Z_1 values with $0 \le Z_1 \le 0.2$ and is proportional in non- rotating solid at high impedance parameter values of Z_1. The effects of rotations in an initially stressed and non-stressed solid on the Rayleigh wave velocity against the impedance parameter Z_1 are shown in fig.4 to fig.6.

FIGURE4: Impedance parameter Z_1 versus square velocity of Rayleigh wave for



P=2.

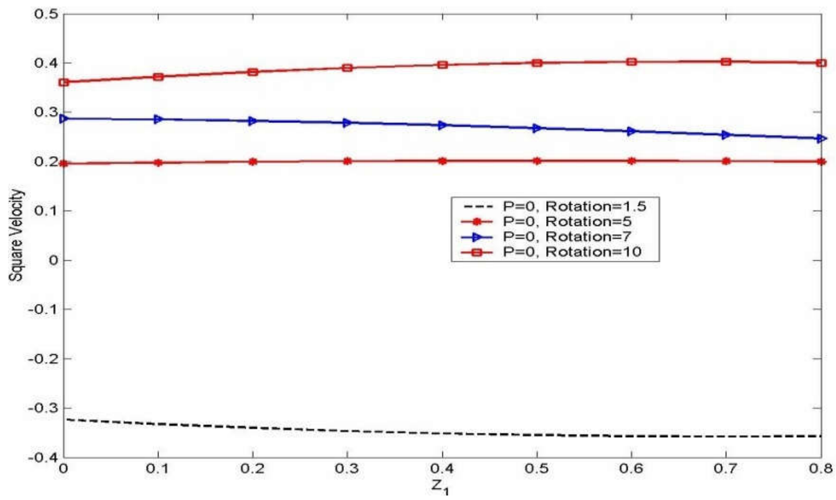


FIGURE 5: Impedance parameter Z_1 versus square velocity of Rayleigh wave for $P = 0$.

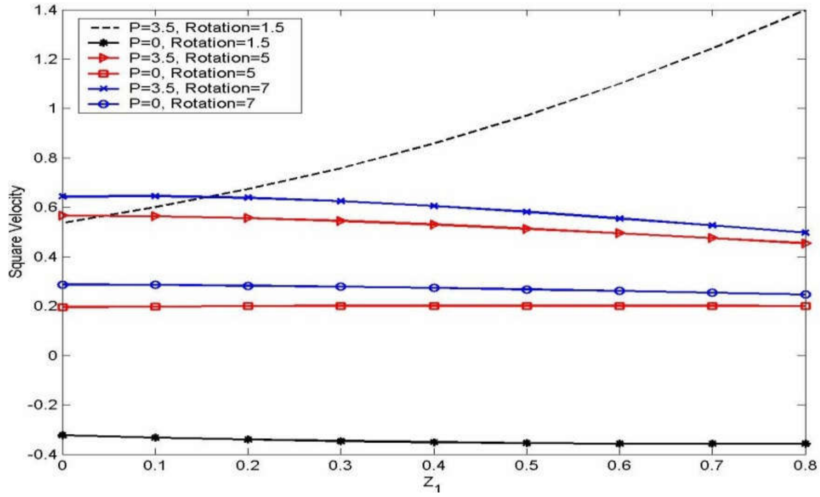


FIGURE 6: Impedance parameter Z_1 versus square velocity of Rayleigh wave for rotating solids.

It is observed that high rotations are proportional to the Rayleigh wave velocity in an initially stressed solid at high impedance parameter Z_1-values. Rayleigh wave velocity is increases with the increasing angular rotation of non-compressed solid. The effects of initial compression in rotating and non-rotating solids on the Rayleigh wave velocity against impedance parameter Z_2 with $0 \le Z_2 \le 1$ are shown in fig.7 to fig.9.

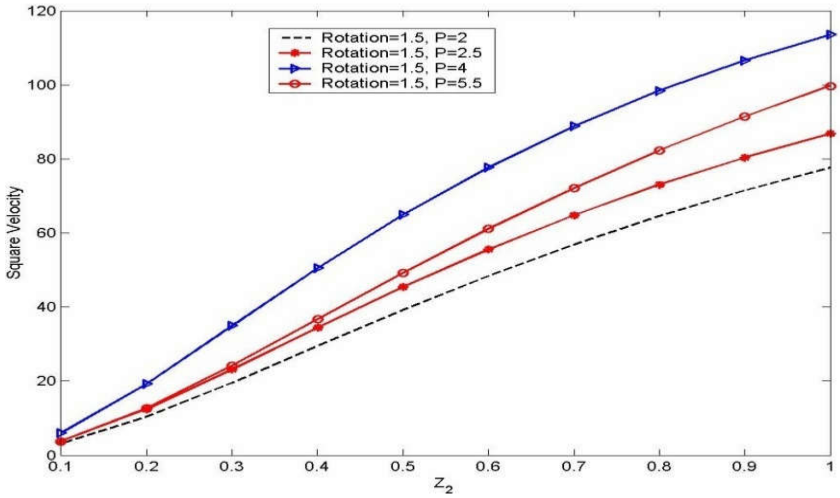


FIGURE 7: Impedance parameter Z_2 versus square velocity of Rayleigh wave for

rotation=1.5.

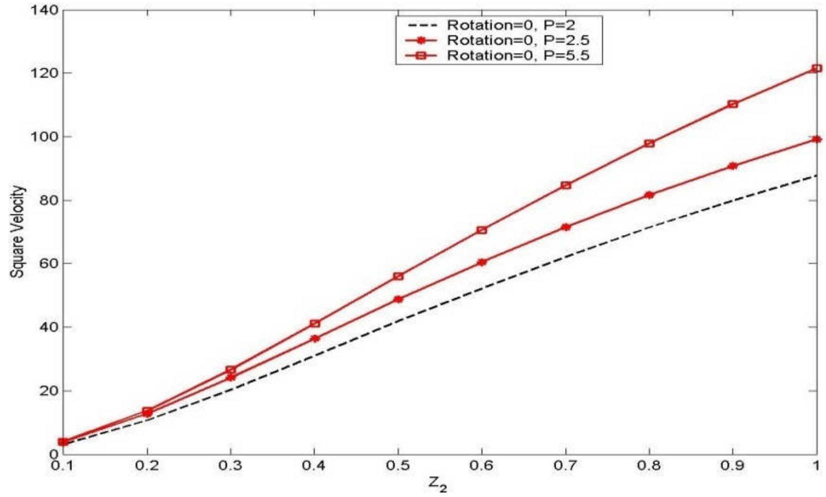


FIGURE 8: Impedance parameter Z_2 versus square velocity of Rayleigh wave for rotation=0.

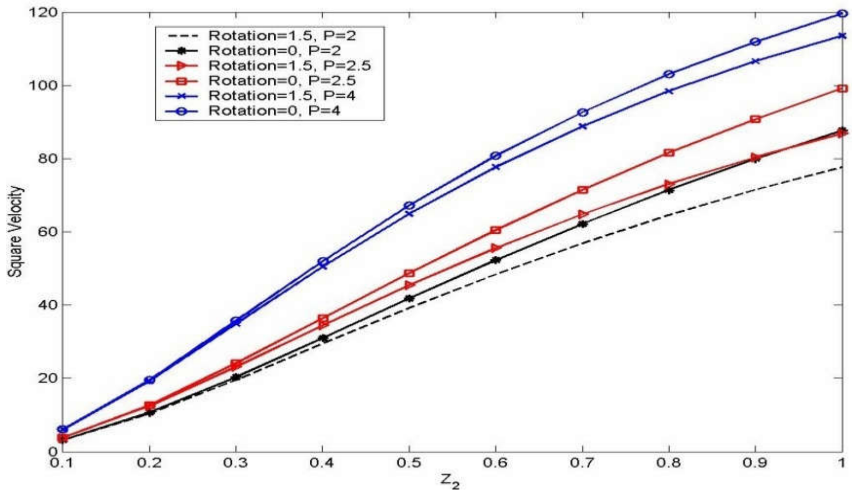
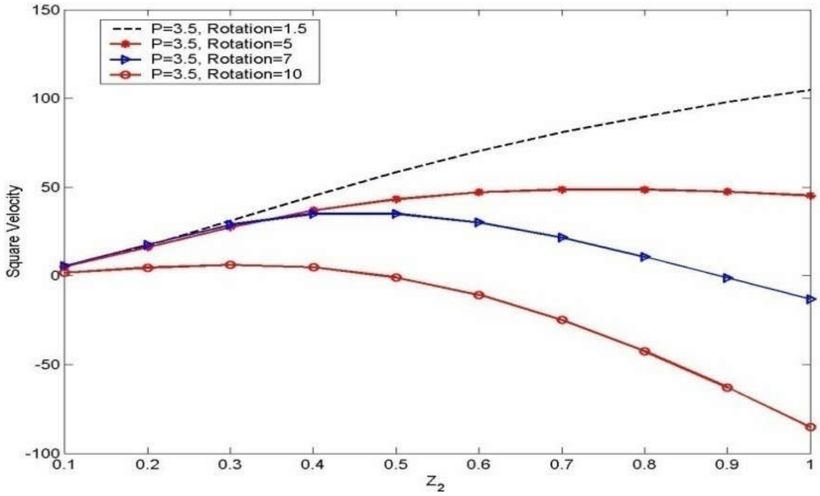


FIGURE 9: Impedance parameter Z_2 versus square velocity of Rayleigh wave for non-rotating, rotating & different initial stresses.

Rayleigh waves in a rotating and initially stressed solid are vanishes at low impedance parameter values $Z_2 = 0.1$. Initially stress is proportional to the Rayleigh wave velocity in a non-rotating solid and these waves also vanishes at low impedance parameter value Z_2 = 0.1 and propagate with high speed and high impedance parameter Z_2 = 1.The effects of rotation in an initially stressed and non-stressed solid on the Rayleigh wave velocity against the impedance parameter Z_2 are shown in fig.10 to fig.12.

FIGURE 10: Impedance parameter Z_2 versus square velocity of Rayleigh wave



for P=1.5.

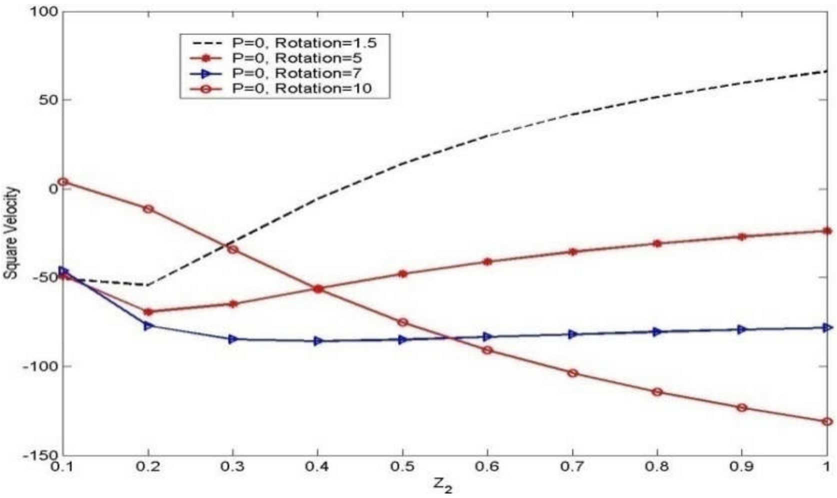


FIGURE 11: Impedance parameter Z_2 versus square velocity of Rayleigh wave for $P = 0$.

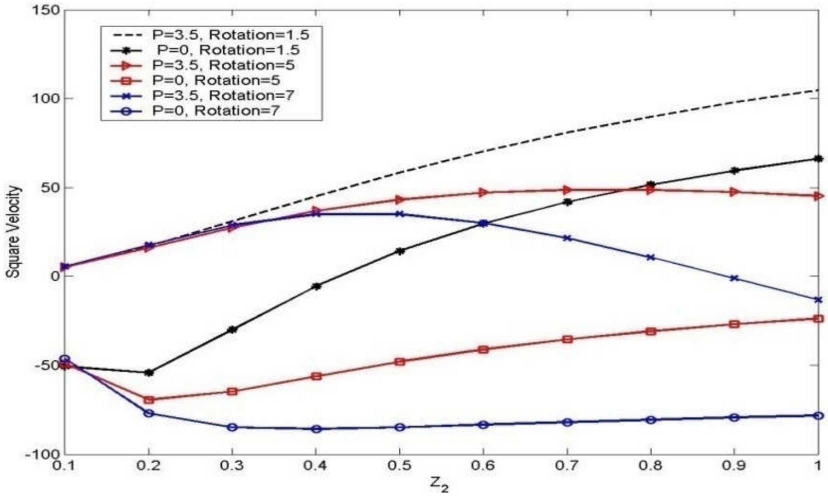


FIGURE 12: Impedance parameter Z_2 versus square velocity of Rayleigh wave for different rotations & different initial stresses.

From these figures, one can observed that the Rayleigh wave velocity is decreases with the increasing angular rotations of initially stressed solids. Also the Rayleigh wave velocity is decreases with the increasing angular rotations in a non-stressed solid at high impedance Z_2 parameter values with $0.7 \leq Z_2 \leq 1$. The waves in a rotating, initially stressed solid are vanishes at low impedance parameter Z_2 with $Z_2 = 0.1$ and having high speed at high impedance parameter Z_2 with $Z_2 = 1$.

7 CONCLUSION

In this study, the effects of angular rotation and initial stress on Rayleigh waves in an orthotropic elastic solid in its gravity field with impedance boundary conditions are considered. The basic governing equations are solved by using traditional techniques to obtain the dispersion relations of Rayleigh waves. From the theoretical computations and a numerical example, we may conclude that:

i) The dispersion relations pertaining to Rayleigh surface waves are derived in orthotropic elastic solid.

ii) Rayleigh surface waves in an orthotropic elastic solid in its gravity field with impedance boundary parameters are influenced by the speed of angular rotation and initial stress of the solid.

iii) Initial stresses of rotating solids are proportional to the Rayleigh wave velocity.

iv) Rayleigh wave velocity in a rotating solids are proportional to the initial stress of the solid and it is inverse proportional to initial stress at low impedance parameters in non-rotating solids.

v) Rayleigh wave velocities are proportional to high rotations of initially stressed solids, while they are proportional to the rotations in non-stressed solids.

vi) Rayleigh wave velocities in rotating and initially stressed solids are vanishes at low impedance parameter $Z_2 = 0.1$ and propagate with high velocities at high impedance parameter $Z_2 = 1$.

The theoretical development of dispersion relations pertaining to Rayleigh type surface waves can be immensely helpful for the researchers who are working in the fields of geophysics, earthquake engineering, seismology and mine engineering. The considered particular numerical example of this article is a model to them for further investigations in the behavior of earth layers under particular boundary conditions.

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