Development of the testıng algorıthms for ıncreasıng of the measurement accuracy

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ABSTRACT
Implementation test algorithms to effectively increase the accuracy of measurement by using simple additive and multiplicative tests have been developed to enable and determine the measured values of non-electrical quantities by the results of additional test measurements used for identification of nonlinear transformation functions of information measurement systems. For the maximum simplification of processing algorithm of the measurement results of the measurement system formulated the problem of determination of the optimal set of additive and combined tests. At the same time was solved of the problem of minimization the number of constant components of tests which used for identification of with enough non-linearity of information-measurement system.

KEYWORDS
Measurement results, test algorithms, information-measurement tested system, main test equations, transformation function, multiplicative tests, additive tests.

INTRODUCTION
As it is known complication of the test algorithms depends on nonlinearity of transformation function (TF) of the measurement system (MS) is and quality of the used test algorithms. During the identification of TF of the MS on base of the certain additional number of measurements are used additive and multiplicative tests with optimal contents. It allows to get the main test equations (MTE) (n-1)-th degree in relation to measured quantity *x*. Accordingly with this the procedure of the determination of the quantity x_{cal} at information-measurement tested system (IMTS) on additional measurements on base of used tests becomes very complicated. Because, in this paper investigated the problem of the development of the new types of the test methods which increase the measurement accuracy of the IMS with non-linear TF. Due to non replaced features this method may be used for measuring of the electrical quantities and also may is widely implemented mainly for measuring of the non electrical values [1-4]. The wide practical and respectable importance of this method have made necessary of the theoretical development of it and requirement on development of the seriouse methodic sinthesis for the IMTS. For the maximum simplification of processing algorithm of the measurement results (MR) of the MS formulated the problem of determination of the optimal set of additive and combined tests. At the same time was solved of the problem of minimization the number of constant components of tests which used for identification of with enough non-linearity of information-measurement system (IMS).

Main test equation of the IMS with non-linearity conversion function

Implementation of the considered test algorithms that used in IMTS in task for increasing of accuracy of MR for the wide classes electrical and particularly for non electrical quantities assumes an method of obtaining the MTE of the IMTS at one unknown - measuring quantity *x* [5]. As it is known, the using of the tests only on base of simple multiplicative and additive tests converts IMTS's MTE to identity [6-8]. So, necessary condition of the existence of MTE is the realization of set of simple multiplicative and additive tests in IMTS. For obtain minimum degree of MTE in relation to *x* are necessary perform *n-1* number of tests one type and one type of another test.

Creating the IMTS via a set of simple additive, multiplicative and combined tests and on base only combined tests has number of features. It relates with conditions of performing the algorithms of MR processing in computing device (Cdev). Let's analyze the MTE of the IMTS.

$$y_0 = \sum_{j=1}^{n} y_j \frac{\prod\limits_{1\le в\le n,\ в\ne j}\left[A_g(x)-x\right]}{\prod\limits_{1\le в\le n,\ в\ne \ell,\ \ell=j}\left[A_g(x)-A_\ell(x)\right]}. \tag{1}$$

Values of parameters of TF of IMTS are detemined from following expressions:

$$\begin{aligned} a_1 = {} & y_1 \frac{A_2(x)A_3(x)....A_n(x)}{\prod\limits_{2\le в\le n}\left[A_в(x)-A_1(x)\right]} + y_2 \frac{A_1(x)A_3(x)....A_n(x)}{\prod\limits_{1\le в\le n,\ \ в\ne 2}\left[A_в(x)-A_2(x)\right]} + ... \\ & + y_j \frac{A_1(x).....A_{j-1}(x)A_{j+1}(x)....A_n(x)}{\prod\limits_{1\le в\le n}\left[A_в(x)-A_j(x)\right]} + ... + y_n \frac{A_1(x)....A_{n-1}(x)}{\prod\limits_{1\le в\le n-1}\left[A_в(x)-A_n(x)\right]} + ... \end{aligned} \tag{2}$$

....

$$a_i = \frac{1}{\underset{1\le l\le в\le m}{\Pi}\left[A_в(x)-A_l(x)\right]}\sum_{o=1}^{m}(-1)^{ш+o}\, y_j\left\{\Pi\left[A_в(x)-A_l(x)\right]\cdot\sum_{\substack{1\le l,в\le n,\\ l\ne j,\ в\ne j}}^{C_n^{n-1}} A_{\alpha 1}(x)A_{\alpha 2}(x)\ldots A_{\alpha n-i}(x)\right\};$$

....

$$a_n = (-1)^{n+1}\left\{\frac{y_1}{\underset{2\le в\le n}{\Pi}\left[A_в(x)-A_1(x)\right]}+\frac{y_2}{\underset{1\le в\le m,\ в\ne 2}{\Pi}\left[A_в(x)-A_2(x)\right]}+\ldots y_j\frac{1}{\underset{1\le в\le n-1}{\Pi}\left[A_в(x)-A_n(x)\right]}\right\}.$$

As seen from (1), at $a_i \ne 0$ $(i=1,\ldots\ldots n)$ and when used only of the combined tests in IMTS, MTE does not convert to identity in relation to measured quantity x.

Really, for identification of TF of IMS needs to perform additional n measurements of the following combined tests in system:

$$A_в(x) = k_в x + \theta_в, \qquad (в = 1,\ldots\ldots, n). \tag{3}$$

After writing the combined test (3) and values of mesurement results of them in expression (1) we will got following relation for IMTS:

$$y_0 = \sum_{j=1}^{n} y_j \frac{\underset{1\le в\le n,\quad в\ne j}{\Pi}\left[x(k_B-1)+\theta_в\right]}{\underset{1\le в\le n,\ в\ne l,\ l=j.}{\Pi}\left[x(k_в-k_l)+(\theta_в-\theta_l)\right]}. \tag{4}$$

As seen from expression (4), at $\theta_в \ne 0,\ \ k_в \ne 0,\ k_в \ne 1$ and when used only of the combined tests in IMTS MTE has at least the one solve on regarding to measured quantity *x*. It is necessary to note that realization of the accurate values of quantities $k_в$ in relation to given values of constant components of additive tests $\theta_в$ practically is difficult.

That is why, during forming the combined tests in form (3) in IMTS the parameter $k_в$ stays constant ($k = const.$) and a value of parameter $\theta_в$. is changed. According with this, expression (4) will write as following:

$$\begin{aligned} y_0 = {} & y_1\frac{\left[x(k-1)+\theta_n\right]\left[x(k-1)+\theta_{n-1}\right]\ldots\ldots\left[x(k-1)+\theta_2\right]}{(\theta_n-\theta_1)(\theta_{n-1}-\theta_1)\ldots\ldots(\theta_2-\theta_1)} - \\ & - y_2\frac{\left[x(k-1)+\theta_n\right]\left[x(k-1)+\theta_{n-1}\right]\ldots\ldots\left[x(k-1)+\theta_3\right]\left[x(k-1)+\theta_1\right]}{(\theta_n-\theta_2)(\theta_{n-1}-\theta_2)\ldots\ldots(\theta_3-\theta_2)(\theta_2-\theta_1)} + \ldots\ldots \\ & + y_j\frac{\left[x(k-1)+\theta_n\right]\ldots\left[x(k-1)+\theta_{j-1}\right]\left[x(k-1)+\theta_{j-1}\right]\ldots\ldots\left[x(k-1)+\theta_1\right]}{(\theta_n-\theta_1)\ldots(\theta_{j-1}-\theta_j)(\theta_{j-1}-\theta_j)\ldots\ldots(\theta_1-\theta_j)} + \\ & + \ldots + y_n\frac{\left[x(k-1)+\theta_{n-1}\right]\ldots\ldots\left[x(k-1)+\theta_1\right]}{(\theta_{n-1}-\theta_n)(\theta_{n-2}-\theta_n)\ldots(\theta_1-\theta_n)}. \end{aligned} \tag{5}$$

Numerators of all terms of expression (5) are polinomials (*n-1*) degree in relation to measuring quantity *x*, and coefficients $\mu_j = F(\theta_{1,\ldots,}\theta_n)$, at denominators don't depend from *x*. So, for all values / $a_i \ne 0$ ($i = 1,\ldots,n$) and when only used combined tests in form $A_в(x)$, MTE will be consist of equations system of (*n-1)* degree in relation to measured quantity x. It note that if given additive and multiplicative test system, the system on combined tests corresponding with it may be obtain via all possible summing and subtracting the tests included to initial system and pairs of $\theta_в$, quantities and then in summing and subtracting results take part the measurement quantity *x*.

We will consider concrete task for clarifying the above mentioned determination. Assume, performs test algorithm for increasing of measurement accuracy on base the (*n-1*) additive and multiplicative tests for identification TF IMTS that approximated by polinomial with (*n-1*) degree. Then in given IMTS can be created a number of possible combined tests that (without compromising the integrity) are determined by following relation.

$$\alpha = \frac{(n-1)(n+8)}{2} \tag{6}$$

We show that realization the system with S^* tests does not convert to identity MTE IMTS with TF following form of the initial MS that consist of *(n-1)* additive, of one multiplicative and of α combined tests,

$$y = \sum_{i=1}^{s^*} a_i x^{i-1}, \qquad (7)$$

where $s^* = \alpha + n$..
MRE obtained in the result of realization in IMTS will has following form

$$y_\circ = \sum_{i=1}^{s^*} y_j \frac{\prod\limits_{1 \le в \le s^*,\ в \ne j} [A_в(x) - x]}{\prod\limits_{\substack{1 \le в \le s^*, \\ в \ne \ell,\ \ell = j.}} [A_в(x) - A_\ell(x)]} \qquad (8)$$

or we knowing $S^* = \alpha + n$,, we get from (8)

$$y_\circ = \sum_{i=n+1}^{s^*} y_j \frac{\prod\limits_{n \le в \le s^*,\ 1 \le \ell \le n-1, в \ne j} [A_в(x) - x] \prod \theta\ell}{\prod\limits_{\substack{n+1 \le в \le s^*,\ в \ne j \\ \ell = j}} [A_в(x) - A_\ell(x)] \prod\limits_{\ell = j} [A_n(x) - A_\ell(x)] \prod\limits_{1 \le в \le n-1,\ \ell = j} [A_в(x) - A_\ell(x)]} +$$

$$+ y_n \cdot \frac{\prod\limits_{n < в \le s^*,\ 1 \le \ell \le n-1} [A_в(x) - x] \prod \theta\ell}{\prod\limits_{\substack{n+1 \le в \le s^*, \\ \ell = j}} [A_в(x) - A_n(x)] \prod\limits_{1 \le в \le m-1} [A_n(x) - A_в(x)]} + \qquad (9)$$

$$\sum_{i=1}^{n-1} y_j \frac{\prod\limits_{n \le в \le s^*,\ 1 \le \ell \le n-1, в \ne j} [A_в(x) - x] \prod \theta\ell}{\prod\limits_{\substack{n \le в \le s^*, \\ 1}} [A_в(x) - A_\ell(x)] \prod\limits_{\le \ell \le n-1} \prod\limits_{\substack{1 \le в \le n-1, \\ \ell = j, \ell = j}} [\theta_в \theta_\ell]} = B + C^* + Q^*.$$

Taking into account expression (6), we shall write summand C^* that includes x and has less multipliers than in B and Q^*, summands of the numerator and denominator of expression (9) as following:

$$C^* = \frac{\prod\limits_{1 \le в \le n-1} [x(k-1) + \theta_в] \prod\limits_{1 \le в \le n-1} [xk + \theta_в]}{\prod\limits_{1 \le в \le n-1} (-\theta_в) \prod\limits_{1 \le в \le n-1} (x + \theta_в) \prod\limits_{1 \le в \le n-1} [-(x + \theta_в)]}.$$

$$\frac{\prod\limits_{1 \le \ell \le n-2.\quad \ell < в < n-1} [x + (\theta_в + \theta_в)]}{\prod\limits_{1 \le \ell \le n-2. \ell < в < n-1} [x(2-k) + (\theta_\ell + \theta_в)]}. \qquad (10)$$

That can be shown by analyzing the expression (10) that for arbitrary $\theta_в \ne 0, k \ne 0, k \ne 1$ a degree of polinimial in numerator more than degree of polinomial in denominator on *(n-1)*. Really, if we take into account multipliers d^* and c that have a measured quantity corresponding in numerator and in denominator, we will get a following expression:

$$d^* - c = \left[3(n-1) + \frac{n-1}{2} \cdot n\right] - \left[2(n-1) + \frac{n-1}{2} \cdot n\right] = n - 1.$$

So, it is clear that in numerator and in denominator of expression (10) do not exist *(n-1)* common multipliers that include x. We can make conclusion that equation (10) does not convert to identity in area where perform search the parameters $\theta_в$ and k. If take into account (9) same way for all $a_i \ne 0 \left(i = 1, \ldots, s^*\right)$ we will get that MRE IMTS does not convert to identity in relation to measured quantity x on the possible additive combinated of (n-1) additive and one multiplicative tests and α of combined tests obtained based on them. As shown, the simple additive and multiplicative set of tests, the possibility of creating combinateds of tests are very different. However, application of the simplest tests in forms $kx + \theta$ or $kx - \theta$, that don't expose the problems of forming at practical implementation of IMTS is more appropriate.

Determination of the optimal sets of tests for identification of the non-linear transducing function of the IMS

As previously noted, the minimal number of MTE's degree during the implementation in IMTS of the test algorithm based on a simple additive and multiplicative measurements is achieved when using in system the (n-1) tests of one type and one test another type [9-15]. And, as a rule, the main advantage is given to additive tests, which are formed a fairly simple way at measuring the electrical and non-electrical quantities. In this case, according to (1) the total form of the MTE can be described as follows:

$$y_0=\frac{y_1x(k-1)\theta_{n-1}\dots\theta_2}{[x(k-1)-\theta_1](\theta_{n-1}-\theta)\dots(\theta_2-\theta_1)}-\frac{y_2x(k-1)\theta_{n-1}\dots\theta_3\theta_1}{[x(k-1)-\theta_2](\theta_{n-1}-\theta_2)\dots(\theta_3-\theta_2)(\theta_2-\theta_1)}+$$

$$+\dots+\frac{y_{n-1}(-1)^{n-2}x(k-1)\theta_{n-2}\dots\theta_1}{[x(k-1)-\theta_{n-1}](\theta_{n-1}-\theta_{n-2})\dots(\theta_{n-1}-\theta_1)}+\dots+\frac{y_n(-1)^{n-1}\theta_{n-1}\dots\theta_1}{[x(k-1)-\theta_{n-1}]\dots[x(k-1)-\theta_1]} \tag{11}$$

In given system instead of *(n-1)*-th additive test $x+\theta_{n-1}$, of the simple *n* additive and multiplicative tests (in the next - the initial system of tests) we carry out the combinated test:

$$A_1(x)=kx+\theta_1.$$

Then, the expression (11) will be the following form:

$$y_0=\frac{y_1x(k-1)[x(k-1)+\theta_1]\theta_{n-2}\dots\theta_2}{[x(k-1)-\theta_1][x(k-1)](\theta_{n-2}-\theta_1)\dots(\theta_2-\theta_1)}-\frac{y_2x(k-1)[x(k-1)+\theta_1]}{[x(k-1)-\theta_2][x(k-1)+\theta_2-\theta_1]}+$$

$$\frac{\theta_{n-2}\dots\dots\theta_3\theta_1}{(\theta_{n-2}-\theta_2)\dots(\theta_3-\theta_2)(\theta_2-\theta_1)}+\dots\dots+\frac{y_{n-1}(-1)^{n-2}x(k-1)\theta_{n-2}\dots\theta_1}{(-\theta_1[x(k-1)+\theta_1-\theta_{n-2}]\dots\dots x(k-1))}+ \tag{12}$$

$$+y_n\frac{(-1)^{n-1}[x(k-1)+\theta_1]\theta_{n-2}\dots\theta_1}{(-\theta_1)[x(k-1)-\theta_{n-2}]\dots\dots[x(k-1)-\theta_1]}.$$

From (12) get that in case *n=2m* and when consecutive replaced the additive tests $x+\theta_в$ of the initial system, with the combined tests $kx+\theta_{n-в}$ polimonials degree of summands in numerators increases on (*n-1*), polinomials degree of summands of combined and multiplicative tests according to measurements values in denominator decreases to zero. At the same time in denominators of summands corresponding with to measurements of the usual tests, polynomials degree increases as long as, the additive tests in initial test systems will be not replaced with the combined tests.

We will carry out shown replacement of additive tests on combined tests in frame of the initial system. It should be noted, that in a certain value of ratio of the additive and the combinated tests d^* degree of at least of one of the polinomials that is in the numerator of the summands is increased, and degree of at least of one polinomials is in the denominators is decreased and reaches the degree *c*, so that is:

$$\begin{matrix} d^*=c, & & n=2m+1; \\ & \text{at} & \\ d^*=c-1=c^*, & & n=2m; \end{matrix} \tag{13}$$

If additive and combinated of tests realized on IMTS equal to each other, (at $n=2m+1$) or (at $n=2m$) the number of the additive tests equals to sum of multiplicative and of combined tests, as seen from (12) the conditions (13) are fulfilled. Furthermore, at ratio of the quantities $\theta_в$, that include the new test system the initial MTE transform to such the equation which degree is determined by the value c (at $n=2m+1$) or by values c^* (at $n=2m$) in relation to quantity x. Taking into account the above, we form the organizing principle of the optimal combinated of the additive, multiplicative and combinated tests which allows to get minimal degree of MTE at implementation in MS on equal other terms in relation to *x*. At implementation the *m* number of combined tests TF of initial MS and m number of additive tests and one multiplicative test (at $n=2m+1$), and additive components which are subject to conditions $\theta_в-\theta_{в-1}=\theta_1$, in all $a_i\neq 0(i=1,\dots\dots,n,$ where $n==2m$ or $2m+1=n)$ cases MTE has the degree *m* in relation to the measured quantity x. Let's prove this assumption.

It get that at arbitrary values of $\theta_в(1\le в\le n,\ \theta_{m+1}=0,\ \theta_1=\theta_{m+2},\theta_2=\theta_{m+3,\dots\dots,}\theta_\lambda=\theta_{(m+1)+\lambda,\dots})$ in summand the corresponding to measurement of multiplicative tests s1, the numerator and denominator don't have common multipliers that contained *x*. Indeed, in (14) for the summand s_1, exists following relation s_1^*, which has in numerator and in denominator *x*:

$s_1=\mu_{m+1}\cdot s_1^*$,

$$\mu_{m+1} = (-1)^m \frac{\prod\limits_{1\le в\le m}(\theta_в)}{\prod\limits_{m+2\le в\le n}(\theta_в)}$$

Where (15)

$$s_1^* = \frac{\prod\limits_{m+2\le в\le n}[x(k-1)+\theta_в]}{\prod\limits_{1\le в\le m}[x(k-1)+(\theta_{m+1}-\theta_в)]} = \frac{\prod\limits_{m+2\le в\le n}[x(k-1)+\theta_в]}{\prod\limits_{1\le в\le m}[x(k-1)-\theta_в]}.$$

Let's determine from (14) one of the conditions that fulfill the (13) - ratio between number of multipliers d^* in numerator and number of multipliers c in denominator which it have measuring quantity *x* at case (16), i.e. ratio between the number of additive tests ($m = в^*$) and number $(n-m)$ of combined tests including the multiplicative test kx.

$$\text{at } \frac{d^*}{c} = 1, \quad n = 2m+1; \tag{16}$$

$$\text{at } \frac{d^*}{c-1} = 1, \quad n = 2m.$$

Assume, $n = 2m.$ Then we will get the following expression taking it into account that $в^* = m,$ in the expression (15):

$$\frac{\prod\limits_{m+2\le в\le n}[x(k-1)+\theta_в]}{\prod\limits_{1\le в\le m}[x(k-1)+(\theta_{m+1}-\theta_в)]} = \frac{x^{m-1}\cdot R_{m-2} + x^{m-2}\cdot R_{m-3} + \dots + R}{x^m \cdot r_{m-1} + x^{m-1} r_{m-2} + \dots\dots + r}. \tag{17}$$

where, $\frac{d^*}{c-1} = 1.$

Similarly, if $n = 2m+1,$ then at case $в^* = m$ and $n-m = m+1,$ we get following:

$$\frac{\prod\limits_{m+2\le в\le n}[x(k-1)+\theta_в]}{\prod\limits_{1\le в\le m}[x(k-1)+(\theta_{m+1}-\theta_в)]} = \frac{x^m \cdot R_{m-1} + x^{m-1}\cdot R_{m-2} + \dots + R}{x^m \cdot r_{m-1} + x^{m-1} r_{m-2} + \dots\dots + r}. \tag{18}$$

Where

$$\frac{d^*}{c} = 1.$$

In other equal conditions we will show that number of multipliers which have *x* and mutually is not reduced in numerator and in denominator of summand B_1 in expression (14) not more than such multipliers in summand s_1. Taking into account that the depending of maximum degree of quantity *x* in mathematical model (MM) $(a_i \ne 0)$ that has been adopted for TF IMS, is pair or not in MM realised one of the tests systems, in accordance with the terms (13): in case _ $n = 2m(m = в^*)$:

$$\begin{cases} A_1(x) = x + \theta_1 \\ \dots\dots\dots\dots\dots\dots \\ A_m(x) = x + \theta_m \\ A_{m+1}(x) = k\cdot x \\ A_{m+2}(x) = k\cdot x + \theta_{m+2} \\ \dots\dots\dots\dots\dots\dots\dots \\ A_n(x) = k\cdot x + \theta_{n-1} \end{cases} \tag{19}$$

At _ $n = 2m+1$

$$\begin{cases} A_1(x) = x + \theta_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_m(x) = x + \theta_m \\ A_{m+1}(x) = k \cdot x \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_n(x) = k \cdot x + \theta_n \end{cases} \quad (20)$$

For summand B1 from the expression (14) we will get:

$$B_1^* = \frac{\Pi_{m+1 \le в \le n}\left[x(k-1) + \theta_в\right]}{\Pi_{m+1 \le в \le n}\left[x(k-1) + (\theta_в - \theta_m)\right]} = \frac{x^m \cdot r_{m-1} + x^{m-1} \cdot r_{m-2} + \ldots. + r}{x^m \cdot \ell_{m-1} + x^{m-1} \ell_{m-2} + \ldots\ldots + \ell}, \quad (21)$$

Where $B_1 = \mu_m \cdot B_1^*$; $n = 2m$;

$$\mu_m = y_m \cdot \Pi_{1 \le в \le b-1}(\theta_в) \cdot \frac{1}{\Pi_{1 \le в \le m-1}(\theta_в - \theta_m)}, \quad (22)$$

Or

$$B_2^* = \frac{\Pi_{m+2 \le в \le n}\left[x(k-1) + \theta_в\right]\left[x(k-1)\right]}{\Pi_{m+1 \le в \le n-1}\left[x(k-1) + (\theta_в - \theta_m)\right]} = \frac{x^m \cdot r_{m-1} + x^{m-1} \cdot r_{m-2} + \ldots. + r}{x^m \cdot \ell_{m-1} + x^{m-1} \ell_{m-2} + \ldots\ldots + \ell}, \quad (23)$$

Where $n = 2m + 1.$

We get from (21) and (23) the expressions that in both cases: ratio $\frac{d^*}{c}$ for summand B_1 and also for S_1 is provided by formula (16). Now we will determine necessary relation between quantities $\theta_в (в = 1, \ldots, m)$, in case when number of multipliers in numerator and in denominators of summands A and D are not more than number of inreducible multipliers in summands B_1^* and S_1^* that have measured quantity *x*.

Consider the following expressions:

$$A = \sum_{j=1}^{m-1} \mu_j \Pi_{m+1 \le в \le n}\left[x(k-1) + \theta_в\right] \cdot \frac{1}{\Pi\left[x(k-1) + (\theta_в - \theta_j)\right]} =$$

$$= \sum_{j=1}^{m-1} \mu_j \frac{\Pi_{2m-j < в \le 2m}\left[x(k-1) + \theta_2\right] \Pi_{m+1 \le в \le 2m-j}\left[x(k-1) + \theta_в\right]}{\Pi_{m+1 \le в < в Бj+m+12m}\left[x(k-1) + (\theta_в - \theta_j)\right] \Pi_{m+1+j \le в \le m}\left[x(k-1) + (\theta_в - \theta_j)\right]} = \sum_{j=1}^{m-1} \psi_j \varepsilon_j, \quad (24)$$

Here

$$\mu_j = \prod_{1 \le b \le m, b \ne j} (\theta_b) \frac{1}{\prod_{1 \le b \le m, b \ne j} (\theta_b - \theta_j)}; \qquad n = 2m; \quad (25)$$

$$D=\sum_{\lambda=m+2}^{n-1}\mu_\lambda\frac{\prod\limits_{m+1\le\theta\le n,\theta\le\lambda}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{1\le\theta\le m}\left[x(k-1)+(\theta_\lambda-\theta_\theta)\right]}=y_n(-1)^{n-1}_{\mu_n}\frac{\prod\limits_{m+1\le\theta\le n-1}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{1\le\theta\le m}\left[x(k-1)+(\theta_{m-1}\theta_\theta)\right]}=$$

$$=y_n(-1)^m\frac{\mu_n}{\prod\left[x(k-1)+(\theta_{\lambda+1}-\theta_\theta)\right]}\cdot\frac{\prod\limits_{m+1\le\theta\le 2m-1}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{1\le\theta\le m-1}\left[x(k-1)+(\theta_{m-1}\theta_\theta)\right]}+ \tag{26}$$

$$+\sum_{\lambda=m+2}^{n-1}\frac{\mu_\lambda\prod\limits_{\lambda-(m+1)\le\theta\le n,\theta\ne\lambda}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{\lambda-(m+1)<\theta\le m}\left[x(k-1)+(\theta_\lambda-\theta_\theta)\right]}\cdot\frac{\prod\limits_{m+1\le\theta\lambda}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{\lambda\le\theta\le\lambda-(m+1)}\left[x(k-1)+(\theta_\lambda-\theta_\theta)\right]}=\sum_{\lambda=m+2}^{n-1}\alpha_\lambda\cdot\varphi_\lambda+\alpha_n\varphi_n,$$

Where

$$\mu_\lambda=\frac{\prod\limits_{1\le\theta m}(\theta_\theta)\cdot y_\lambda}{\prod\limits_{\substack{m+1\le\theta n\\ \theta\ne\lambda}}\left[\theta_\theta-\theta_\lambda\right]};\qquad \mu_n=\frac{\prod\limits_{1\le\theta\le m}(\theta_\theta)}{\prod\limits_{m+1\le\theta\le n-1}(\theta_\theta-\theta_{m-1})} \tag{27}$$

And when ($n=2m+1,$) expression:

$$A^*=\sum_{j=1}^{m-1}\frac{\mu_j\prod\limits_{m+1\le\theta\le n}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{m+1\le\theta\le n}\left[x(k-1)+(\theta_\theta-\theta_j)\right]}=\sum_{j=1}^{m-1}\frac{\mu_j\prod\limits_{n-j<\theta\le n}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{m+1\le\theta<j+(m+1)}\left[x(k-1)+(\theta_\theta-\theta_j)\right]}\cdot$$

$$\frac{\prod\limits_{m+1\le\theta\le n-j}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{(m+1)+j\le\theta\le n}\left[x(k-1)+(\theta_\theta-\theta_j)\right]}=\sum_{j=1}^{m-1}\psi_j^*\cdot\varepsilon_j^*; \tag{28}$$

$$D^*=\sum_{\lambda=m+2}^{n-1}\frac{\mu_\lambda\prod\limits_{\lambda-(m+1)<\theta\le m,\theta\ne\lambda}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{\lambda-(m+1)<\theta\le m}\left[x(k-1)+(\theta_\lambda-\theta_\theta)\right]}+\frac{\prod\limits_{m+1\le\theta\le n-1}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{1\le\theta\le m}\left[x(k-1)+(\theta_{m-1}\theta_\theta)\right]}=y_n(-1)^m\cdot\mu_n=$$

$$+\sum_{\lambda=m+2}^{n-1}\mu_\lambda\frac{\prod\limits_{\lambda-(m+1)<\theta\le n,\theta\ne\lambda}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{\lambda-(m+1)<\theta\le m}\left[x(k-1)+(\theta_\lambda-\theta_\theta)\right]}\cdot\frac{\prod\limits_{m+1\le\theta<\lambda}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{\lambda\le\theta\le\lambda-(m+1)}\left[x(k-1)+(\theta_\lambda-\theta_\theta)\right]}+ \tag{29}$$

$$+y_n(-1)^m\mu_n\cdot\frac{\prod\limits_{m+1\le\theta\le 2m}\left[x(k-1)+\theta_\theta\right]}{\prod\limits_{\lambda\le\theta\le m}\left[x(k-1)+(\theta_{m1}-\theta_\theta)\right]}=\sum_{\lambda=m+2}^{n-1}\alpha_\lambda^*\varphi_\lambda^*+\alpha_n^*\varphi_n^*,$$

Where

$$\mu_n=\frac{\prod\limits_{1\le\theta\le m}(\theta_\theta)}{\prod\limits_{m+1\le\theta\le n-1}\left[\theta_\theta-\theta_m\right]} \tag{30}$$

The coefficients ψ_j,ψ_j^* in the expressions (24) and (28) is ratio of two polinomials and at arbitrary values of θ_θ and θ_j $(1\le\theta\le m,\quad \theta_1<\theta_2<.....<\theta_m)$, do not have any common multipliers with measured quantity *x*. At the same time, at certain ratio of values θ_θ and θ_j the coefficients $\varepsilon_j,\varepsilon_j^*$ in expressions (24) and (28) will be equal to 1.

The (28) will write as follows:

$$\sum_{j=1}^{n-1}\psi_j^*\varepsilon_j^* = \psi_1^* \frac{[x(k-1)+\theta_1][x(k-1)+\theta_2]\ldots.[x(k-1)+\theta_{m-1}]x(k-1)}{[x(k-1)+(\theta_m-\theta_1)][x(k-1)+(\theta_{m-1}-\theta_1)]\ldots.x(k-1)}+$$

$$+\ldots\ldots+\psi_{m-2}^* \frac{[x(k-1)+\theta_1][x(k-1)+\theta_2]x(k-1)}{[x(k-1)+(\theta_m-\theta_{m-2})][x(k-1)+(\theta_{m-1}-\theta_{m-2})]x(k-1)}+ \tag{31}$$

$$+\psi_{m-1}^* \frac{[x(k-1)+\theta_1]x(k-1)}{[x(k-1)+(\theta_m-\theta_{m-2})]x(k-1)}.$$

Similarly, taking into account (19) and (20) also for expressions (26) and (29) we will get following:

$$\sum_{\lambda=m+2}^{n}\alpha_\lambda^*\varphi_\lambda^* = \alpha_{m+2}^* \frac{[x(k-1)+\theta_{m+1}]}{[x(k-1)+(\theta_{m+2}-\theta_1)]}+\ldots\ldots\ldots+\alpha_{2m}^* \frac{[x(k-1)+\theta_{m+1}]}{[x(k-1)+(\theta_{2m}-\theta_1)]}\cdot$$

$$\cdot\frac{[x(k-1)+\theta_{2m-1}]\ldots\ldots.[x(k-1)+\theta_{m+1}]}{[x(k-1)+\theta_{2m}-\theta_2]\ldots\ldots.[x(k-1)+\theta_{2m}-\theta_{m-12}]}+ \tag{32}$$

$$+\alpha_{2m+1}^* \frac{[x(k-1)+\theta_{2m}]\ldots\ldots.[x(k-1)+\theta_{m+1}]}{[x(k-1)+\theta_{2m+1}-\theta_1][x(k-1)+(\theta_{2m+1}-\theta_2)]\ldots.[x(k-1)+(\theta_{2m+1}-\theta_m)].}$$

Taking into account that the all $\varepsilon_j^* = 1$ and $\varphi_\lambda^* = 1$, in (31) and (32) we will determine the required relationship between the quantities $\theta_в$ from the following system of equations:

$$\begin{cases}\sum_{j=1}^{m-1}\psi_j^*\varepsilon_j^* = \sum_{j=1}^{m-1}\psi_j^* \\ \sum_{\lambda=m+2}^{n}\alpha_\lambda^*\cdot\varphi_\lambda^* = \sum_{\lambda=m+2}^{n}\alpha_\lambda^*\end{cases} \tag{33}$$

Putting the expression (31) and (32) in (33) and taking into account (19) and (20) we will get the searched ratio of constant components $\theta_в$ in the additive and combinated tests:

$$\begin{cases}\theta_{в+1}-\theta_d = \theta_1 \\ \theta_\lambda-\theta_{\lambda-1} = \theta_1,\end{cases} \tag{34}$$

Here $1 \le в \le m, \quad m+2 \le \lambda \le n.$

The condition (34) sets the optimal ratio among values $\theta_в$ for identification of the selected TF for IMS. Test sets (19) and (20) realising with taking into account the condition (34) we will call the optimal sets.
Taking into account condition (34) the expression (14) of the MTE will be in following form:

$$y_0 = \sum_{\lambda j=1}^{m-1}\mu_j \frac{\prod\limits_{n-j<в\le n}[x(k-1)+\theta_в]}{\prod\limits_{m+1\le в<j+m+1}[x(k-1)+(\theta_в-\theta_j)]}+\mu_m \frac{\prod\limits_{m+1<в\le n}[x(k-1)+\theta_в]}{\prod\limits_{m+1\le в\le n}[x(k-1)+(\theta_в-\theta_m)]}=$$

$$\tag{35}$$

$$\mu_{m+1}\frac{\prod\limits_{m+2<в\le n}[x(k-1)+\theta_в]}{\prod\limits_{1\le в\le m}[x(k-1)+(\theta_{m+1}-\theta_в)]}+\sum_{\lambda=m+2}^{n-1}\mu_\lambda \frac{\prod\limits_{\lambda-(m+1)<в\le m, в\ne\lambda}[x(k-1)+\theta_в]}{\prod\limits_{\lambda-(m+1)<в\le m}[x(k-1)+\theta_\lambda-\theta_в]}+$$

Note that in case $n = 2m+1$ the last summand in the expression (35) will take form $y_n(-1)^{n-1}\mu_n$. Then, all set of multipliers in denominators of summands A^{**}, B^{**}, D^{**} – will be equal to number of multipliers in denominator of the summand S^{**}. Let us bring (35) to a common denominator, and we will get a total MTE form IMTS in which realized an optimal set of additive, multiplicative and combined tests. Then we will get for $n = 2m+1$ the following expression:

$$y_0 \cdot \prod_{1\le в\le m}\left[x(k-1)+(\theta_{m+1}-\theta_в)\right] = \sum_{\substack{j=1\\1\le в\le m}}^{m} \mu_j \prod_{n-j<в\le n}\left[x(k-1)+\theta_в\right]\cdot$$

$$\prod_{j<в\le m}\left[x(k-1)+(\theta_{m+1}-\theta_в)\right] + \sum_{\lambda=m+1}^{n} \mu_\lambda \prod_{\lambda-(m+1)<в\le m}\left[x(k-1)+\theta_в\right]\cdot \tag{36}$$

$$\prod_{n-\lambda<в\le m}\left[x(k-1)+\theta_{m+1}-\theta_в\right],$$

Where

$$\mu_j = \frac{m!}{(m-j)!\, j!}(-1)^{j-1}\cdot y_j;$$

$$\mu_\lambda = \frac{[n-(m+1)]!}{(n-\lambda)!\,[\lambda-(m+1)]!}(-1)^{\lambda-1}\cdot y_\lambda .$$

Similarly, will get for $n = 2m$ the following expression:

$$y_0 \prod_{1\le в\le m}\left[x(k-1)+(\theta_{m+1}-\theta_в)\right] = \sum_{j=1}^{m} \mu_j \prod_{n-j<в\le n}\left[x(k-1)+\theta_в\right] \prod_{j\le в\le m}\left[x(k-1)+\theta_{m+1}-\theta_в\right] +$$

$$+\mu_{m+1} \prod_{m+2\le в\le n}\left[x(k-1)+\theta_в\right] + \sum_{\lambda=m+2}^{n-1}\left\{\mu_\lambda \prod_{\lambda-(m+1)<в\le m+1}\left[x(k-1)+\theta_в\right]\cdot\right. \tag{37}$$

$$\cdot \prod_{n-\lambda<в\le m}\left[x(k-1)+(\theta_{m+1}-\theta_в)\right] + \mu_n \prod_{2\le в\le b}\left[x(k-1)+(\theta_{m+1}-\theta_в)\right],$$

where

$$\mu_j = \frac{m!}{(m-j)!\, j!}(-1)^{j-1}\cdot y_j;$$

$$\mu_{m+1} = y_{m+1}\cdot\theta\cdot m\cdot(-1)^m;$$

$$\mu_n = y_n(-1)^{2m-1}\cdot\theta\cdot m\cdot;$$

$$\mu_\lambda = \frac{m!(-1)^{\lambda-1}\cdot y_\lambda}{\nu!(m-1-\nu)!};\quad \nu = \lambda-(m+1).$$

Taking into account that the coefficients $\mu -$ are the independent quantities from x, then all summands in expressions (36) and (37) will be the polynomials in following form:

$$x^c\cdot r_{c-1} + x^{c-1}\cdot r_{c-2} + \dots + r, \quad \text{where } c \le m.$$

So as noted above, in case $n = 2m$ MTE degree is reduced on $m-1$; and at $n = 2m+1$ it is reduced on $\frac{n-1}{2}$ and in both cases it is determined by the degree of the polynomial that arised in the denominator of summand C^{**} during the solution of equation (35). So, if during the operating process of IMTS depending on even and odd of degree (n-1) received for describing MM of TF of initial MS, if additional measurements are performed for the tests that consist of the systems (19) or (20) and their constant components $\theta_в$ are subordinated to relationship (34), degree of obtained MTE will be equal to m. Essentially the following important conclusion can be make out from the foregoing. Realization of IMTS on the basis of additional measurements of optimal combinated of the additive, multiplicative and combined tests on the other equal terms allows to reduce the number of constant components $\theta_в$, which necessary are formed in system and used for the identification TF of the system. At the same time, the number of constant components $\theta_в$ during implementation of IMTS on the basis of additional measurements of simple

additive and multiplicative tests that make up the initial system is reduced on $(n/2)-1-$ if n is even and reduced on $(n-1)/2-$ if n is odd.
Indeed, if

$$\begin{cases} A_1(x) = x+\theta_1 \\ A_{n-1}(x) = x+\theta_{n-1} \\ \dots\dots\dots\dots\dots \\ A_n(x) = kx \end{cases} \quad (38)$$

we compare number of additive constant components $\theta_в$ formed by initial test system and d^* number of these quantities which include tests in the expression (19) and (20).
During the identification TF of initial MS via the additional measurements of optimal set of additive and combined tests taking into account (20) and (34), for MTE we will get following:

$$y_0 = \frac{2y_1(kx+2\theta)}{(kx-\theta)} - \frac{(kx+2\theta)(kx+\theta)}{(kx-\theta)(kx-2\theta)} + y_3\frac{(kx+2\theta)(kx+\theta)}{(kx-\theta)(kx-2\theta)} - y_4\frac{(kx+2\theta)}{kx+\theta} + y_s.$$

The comparison results to unambiguously show the advantages of the use of algorithms based on additional measurements of set of additive and combined tests in IMS with non-linear TF.

The test algorithms for increase the accuracy of measurements based on using of combined tests

If the MM of the real TF of initial MS is determinated, any particular processing algorithm for MR of used tests can be is easy gotten from (36) and (37) expressions. To this end, in known number n of parameters MM of TF of initial MS, depending on it is even or odd, is chosen set of tests from the system of equations (20) or (19). The putting of the values of the tests in appropriate expression (36) or (37), are determined the processing algorithms of MR of system. In most cases of the practical realizations of IMS of non-electric quantities with TF with the second degree polynomial of gives a fairly accurate results. It described as following square trinomial:

$$y = a_{1s} + a_{2s}x + a_{3s}x^2, \quad (39)$$

Here $s = 1,\dots,\ell^* -$ the approximation area.
If taking this into account, the synthesis of the test algorithm on IMS will be realized as follow.
Taking into account (34), for $n=3$ from (20) the optimal set of tests is determined as:

$$\begin{cases} x+\theta \\ kx \\ kx+\theta. \end{cases}$$

If take into account this expression, is determined structure IMTS which allows to implement an additive, multiplicative and the combinated tests at system. Then, at known n=3 and for the established set of tests we determine MTE of the IMTS given in (36):

$$y_0 = \frac{[x(k-1)+\theta](y_1-y_2)+y_3(xk-x-\theta)}{[x(k-1)-\theta]} \quad (40)$$

From the expression (40) is determined processing algorithm a set of tests used in the system according the quantity x:

$$x_{выч.} = \frac{(y_1-y_2)+(y_0-y_3)}{(y_0-y_3)-(y_1-y_2)}\cdot\frac{\theta}{(k-1)}. \quad (41)$$

The computing device IMTS is processed the obtained of the measurements results $y_0,\dots\dots,y_3$ on algorithm (41) and determines the measured quantity x. If the non-linearity of the TF of initial MS is happened as result approximation of it in parts by using polinomials with third, fourth degree and so on, then MTE IMTS will be non-linear in relation to measured quantity x and its solution in general case can be perform by iterative algorithms [16-20] or with via the numerical methods implemented on computers. As an example, from expressions (36) and (37) we will determine MTE IMTS which the TF is approximated in parts by using polinomials with respectively third, fourth degree. In case n=4 in accordance with expressions (19) and (34) for to organize test algorithm IMS must be implemented the following set of tests:

$$\begin{cases} A_1(x) = x+\theta_1 \\ A_{21}(x) = x+2\theta \\ A_3(x) = kx \\ A_4(x) = kx+\theta. \end{cases} \quad (42)$$

Putting the values and measurement results of the tests in expression (37) and taking into account $y_0 = a_{1s} + a_{2s}x = a_{3s}x^2 + a_{4s}x^3$, will get MTE IMTS in following form:

$$x^2(k-1)^2(y_0 - 2y_1 + y_2) - x(k-1)\theta(3y_0 - 2y_1 - y_2 + 2y_3 - 2y_4) + \\ + \theta_2(2y_0 + 4y_1 - 2y_3 - 4y_4) = 0. \quad (43)$$

Similarly, at n=5, taking into account (20) and (34) from expression (36) get the:

$$x^2(k-1)^2(y_0 - 2y_1 + y_2 - y_3 + 2y_4 - y_5) - 3x(k-1)(y_0 - y_2 + y_3 - y_5) + \\ + \theta_2(2y_0 + 8y_1 - 2y_2 - 8y_4 - 2y_5) = 0, \quad (44)$$

Where

$$y_0 = a_{1s} + a_{2s}x + a_{3s}x^2 + a_{4s}x^3 + a_{5s}x^4.$$

The searching area of root of equations (43) and (44) in most cases can be approximately determinated from the function which inverse to the nominal conversion function $y(x)$, at n=4 and n=5.

In this case, the measured quantity x is determined by comparing the results of the inverse transformation y_0. with the two roots obtained by solution of equations (42) and (43).

It should be noted that via inputting additional time by the combined and additive tests can be got the algorithm to increase of the measurements accuracy. Its implementation leads to the MTE IMTS with the non-linear TF to linear MTE and thus the need in solve the equation with high degree in CDev is eliminated. Obtained algorithm, based on approach to the measurements process in $\frac{3n-1}{2}$ clock at $n = 2m$ and in $\frac{3n-1}{2}$ clock at $n = 2m+1$, it which included main measurement (x, y_0) at $(n+1)$ clock as taking into account in operating algorithm of the ordinary IMTS. In this case, additional tests are formed in such way that condition (34) of obtaining set of new tests at $n = 2m$ on the basis of an initial system (19) or at $n = 2m+1$ of the initial system (20) should be fulfilled.

Assume, the approximation polinomial of the TF of initial MS is expressed in the following way:

$$y = a_1 + a_2x + a_2x^2 + a_4x^3.$$

In this case, the MTE IMTS will be in the form (43).

In the system is formed additional test $A_S(x) = kx + 2\theta$. In this case, a new set of tests fulfills a condition (34). Putting the values of the tests $A_1(x), A_2(x), A_4(x), A_5(x)$ in expression (37) we get:

$$y_0[x(k-1)][x(k-1)-\theta] = y_1 2[x(k-1)+2\theta][x(k-1)-\theta] - y_2[x(k-1)+\theta]. \\ [x(k-1)+2\theta] + y_4 2\theta[x(k-1)+2\theta] - y_5 2\theta[x(k-1)-\theta]. \quad (45)$$

The processing algorithms of the MR used tests are determined from (43) and (45):

$$x_{выч.} = \frac{(y_0 - y_5 - y_2 - y_3)}{(y_0 + y_2 + y_3 + y_5 - 2y_1 - 2y_4)} \cdot \frac{\theta}{(k-1)}. \quad (46)$$

Similarly, if $y = a_1 + a_2x + a_3x^2 + a_4x^3 + a_2x^4$ is the approximation polinomial of the TF of initial MS, then the processing algorithm of MR of tests used in CDev will be as follows:

$$x_{выч} = \frac{L_0 \cdot P_0 - L_1 \cdot P_1}{L_1 \cdot P_2 - L_0 \cdot P_3} \cdot \frac{\theta}{(K-1)}, \quad (47)$$

Where

$$A_1(x) = x + \theta, \quad A_2(x) = x + 2\theta, \quad A_3(x) = kx, \\ A(x) = kx + \theta, \quad A_5(x) = kx + 2\theta$$

are tests which are necessary for implementation the identification of TF of given IMS accordance with the terms (13), (20) and (34). So, as has been noted above, the implementation of the developed methods for creating the operating algorithms IMS allow to reach number of new qualities in new modification of the test systems, besides specific advantages for IMTS which are realised based on the simple additive and multiplicative tests. Basic from them are:

- simplification of the work algorithm of CDev which allows to determine value x on based on results of additional measurements of tests used for identification of the non-linear TF of IMS;
- a significant reduction in the number of constant components used tests are necessary for the identification of the non-linear TF in IMS;
- a significant simplification of the linearising procedure of TF IMTS with non-linear TF based on reducing the number of necessary measurements of additional tests.

The investigation of the errors of IMTS which realised on the basis of combinated tests.

The generalized model of the error of IMTS. Due that the real operating characteristics of measurement systems are non-linear, the description of their MM with the required accuracy have on particular importance in theoretical and practical point of view.

The difference with the TF of initial IMS determines the value of the final error. When the mathematical model which cover all range of measurements is quite complex - is in the form with high degree polynomial, this curve is approximated non-linear by parts step by step and lead to simple polynomial [21-25]. The partition of the measuring range of each measuring device with square polynomial starting from zero to its final value make MM much simple and the number of measurements clocks are reduced. In difference to the iteration method here the steps not take be equal, the "iteration" steps are determined by the points of change of sign of values and crisis points of curve of TF. In comparison with the classic and known algorithmic methods the effectiveness of this method has significantly advantage than existing methods. In order to evaluate the obtained results are investigated the final measurement errors of the algorithmic-testing method which is developed to increase the accuracy of measurements [26-29]. For get the generalized formula of the error of IMTS based on analysis of the proposed operation algorithm should be determined the each components. In the range where the real TF of initial IMS approximated by parts the mathematical model of it is considered as a following polinomial:

$$y = a_{1s} + a_{2s}x + a_{3s}x^2 \tag{48}$$

In this case, for the realization of the test algorithm to increase accuracy at measurement system carried out the additional measurements of the $x+\theta$; kx; $kx+\theta$ tests.

The measuring results of the tests and quantity *x* can be described as follows:

$$\begin{cases} y_0 = a_{1sH} + a_{2sH}x + a_{3sH}x^2 \\ y_1 = a_{1sH} + a_{2sH}(x+\theta) + a_{3sH}(x+\theta)^2 \\ y_2 = a_{1sH} + a_{2sH}xk + a_{3sH}(xk)^2 \\ y_3 = a_{1sH} + a_{2sH}(xk+\theta) + a_{3sH}(xk+\theta)^2, \end{cases} \tag{49}$$

Where - a_{1sH}, a_{2sH}, a_{3sH} – are the nominal parameters of the TF of MS.

Taking into account that the real current values of the parameters a_{is} are equal to their nominal values, then results of measurements clocks of the x, $x+\theta$, xk and $xk+\theta$ quantities will have the substantial errors. Taking into account these errors in the system of equations (49) will get the following equations system:

$$\begin{cases} y_0 + \Delta_0 = a_{1s} + a_{2s}x + a_{3s}x^2 \\ y_1 + \Delta_1 = a_{1s} + a_{2s}(x+\theta) + a_{3s}(x+\theta)^2 \\ y_2 + \Delta_2 = a_{1s} + a_{2s}xk + a_{3s}(xk)^2 \\ y_3 + \Delta_3 = a_{1s} + a_{2s}(xk+\theta) + a_{3s}(xk+\theta)^2 \end{cases} \tag{50}$$

here $\Delta_0, \ldots, \Delta_3$ – errors of measurement clocks that lead at output of MS

Taking into account the expression (40) in (50) for the MTE IMTS we will get the following expression:

$$\begin{aligned} &y_0[x(k-1)-\theta] + \Delta_0[x(k-1)-\theta] = y_1[x(k-1)+\theta] + \\ &+\Delta_1[x(k-1)+\theta] - y_2[x(k-1)+\theta] - \Delta_2[x(k-1)+\theta] + (y_3+\Delta_3)[x(k-1)-\theta] \end{aligned} \tag{51}$$

Due to that in (51) in difference from the expression (40) is taken into account the errors measurement clocks, difference between expressions (51) and (40) give us error Δ_{TT}:

$$\Delta_{TT} = [x(k-1)+\theta](\Delta_1 - \Delta_2) + [x(k-1)-\theta](\Delta_3 - \Delta_0). \tag{52}$$

Because the expression (52) is the function that determines the final errors of IMTS, putting the values of tests θ and k, as well as values of errors $\Delta_0, \ldots, \Delta_3$ in their place we will get mathematical models (formulas) for all its components. Then, the expression (52) can write as follows:

$$\Delta_{TT} = \theta[\Delta_1 - \Delta_2 - (\Delta_3 - \Delta_0)] + x(k-1)\cdot[\Delta_1 - \Delta_2 + (\Delta_3 - \Delta_0)]. \tag{53}$$

For the absolute error $\Delta_{gir.}$ leads to input of IMTS will get:

$$\Delta_{gir.} = f_{TT}^{-1}[f_{TT}(x) + \Delta_{TT}] - x = \frac{\Delta_{TT}}{f_{TT}^{1}(x)}, \tag{54}$$

Here $f_{TT}(x) = (y_0 - y_0)[x(k-1)-\theta] + (y_2 - y_1)[x(k-1)+\theta].$

Via differentiation of expression (40) on *x* will get for $f'_{TT}(x)$ the following value

$$f'_{TT}(x) = (k-1)[(y_0 - y_3) - (y_1 - y_2)]. \tag{55}$$

we putting the expressions (53) and (55) in (54) we get:

$$\Delta_{gir.} = \frac{\{\theta[(\Delta_1-\Delta_2)-(\Delta_3-\Delta_0)]+x(k-1)[\Delta_1-\Delta_2+(\Delta_3-\Delta_0)]\}}{(1-k)2\theta\{a_{2SH}+a_{3SH}[(k+1)x+\theta]\}}. \quad (56)$$

If the real TF of the initial MS approximate non linear by parts via third-degree polynomial, then the in identification TF implemented the algorithm (46) and the error $\Delta^*_{TTgir.}$ of IMTS lead to output of the system will be in following form:

$$\Delta^*_{TTgir.} = \frac{\Delta^*_{TT}}{f'_{TT}(x)} = \frac{(\Delta_0+\Delta_2+\Delta_3+\Delta_s-2\Delta_1-\Delta_4)x}{2\theta^2\{2a_{3s}+3a_{4s}[x(k+1)+2\theta]\}} + \\ + \frac{(\Delta_2-\Delta_0+\Delta_3-\Delta_s)}{2\theta(k-1)\{2a_{3s}+3a_{4s}[x(k+1)+2\theta]\}}, \quad (57)$$

Where

$$\Delta^*_{TT} = (\Delta_0+\Delta_s)(z-\theta)+(\Delta_2+\Delta_3)(z+\theta)-2z(\Delta_1+\Delta_4);\ z=x(k-1). \quad (58)$$

It should be noted that a large number of errors Δ_i of measurement clocks is the result of influence the many random factors $\vec{\eta}_1, \vec{C}, \vec{L}$. Therefore, if the components of errors Δ_i are have the same degrees on weight ratios, according to the central limit theorem can be forward the idea about the law of normal distribution of errors. As a result, errors Δ_{TT}, Δ^*_{TT}, $\Delta^*_{TTgir.}$, $\Delta_{gir.}$ in expressions (52), (56), (57) and (58) are random quantities and of their distribution laws can be determined as summ of the distributions of random quantity Δ_i and therefore, they adopted as normal. So, for a complete description need to know the two main aspects: mathematical expectation M_Δ and the dispersion D_Δ.

If take into account the analysis of the expressions (52), (56), (57) and (58), perfomed above, can be noted the main following features of the errors IMTS that realised based on the combinated tests:

1. If the absolute errors Δ_i of measurement clocks have same a mathematical expectations M_Δ with the implementation of algorithms (41) and (46), then the mathematical expectations of errors of IMTS operating on given algorithms will be equal to zero.

2. If the additive constant components of tests $\theta_в$, which used on fulfill relation condition (34) between tests in given system of tests, will approach to infinitely small value, then the absolute error of the IMTS, lead to input of the system, will approach to infinity. This feature of the error directly is obtained from the expressions (56) and (58).

At the same time it is clear that at $k \to 1$ the errors $\Delta_{gir.}$ and $\Delta^*_{TTgir.}$ are approached to infinity. Thus, we can conclude that main features of errors of IMTS where are realised algorithms of improve the measurement accuracy based on the simple additive and multiplicative tests are justified for IMTS operating based on optimum set of the additive, multiplicative and combined tests. When the errors of measurement clocks are independed for the dispersion of errors of IMTS where are realised algorithms (41) and (46), respectively we get:

$$\sigma^2_{\Delta_{TT}} = \sigma^2_{\Delta_0}[x(k-1)-\theta]^2+\sigma^2_{\Delta_1}[x(k-1)+\theta]^2+ \\ +\sigma^2_{\Delta_2}[x(k-1)+\theta]^2+\sigma^2_{\Delta 3}[x(k-1)-\theta]^2; \quad (59)$$

$$\sigma^2_{\Delta^*_{TT}} = (\sigma^2_{\Delta_0}+\sigma^2_{\Delta_5})(z-\theta)^2+(\sigma^2_{\Delta_3}+\sigma^2_{\Delta_2})(z+\theta)^2+4z^2(\sigma^2_{\Delta_1}+\sigma^2_{\Delta_4}), \quad (60)$$

Here, $\sigma_{\Delta i}$ – is the respective average square deviation of the measurement errors; $z = x(k-1)$. As seen from expression (59) and (60) dispersion of error Δ_{TT} of MTE in IMTS where are realised the test algorithms (41) and (46) to increasing the accuracy of measurements, compared with dispersion of one clock of measurement is increased. The analysis of error in IMTS where is realised the test algorithm in form (41), will consider on example of evaluation of errors of that test system. It should be noted that evaluation of errors of IMTS where is realised the algorithm (46), can be carried out similarly as in case of using algorithm (41) and in this case the analysis methods remain unchanged [30, 31]. Investigation of the metrological characteristics of IMTS shown that to error of MR of these types systems most influence the following components:

- the component of error caused due to realisation of the constant components θ and κ of additive and multiplicative tests;
- uncorrelated component of the static error of IMTS;
- the dynamic component of the error;
- the error component caused by inadequacy of MM which adopted for TF of initial MS.

Conclusions

The above-mentioned positive qualities once again prove the advantage of using of IMTS realized based on the optimal set of additive, multiplicative and the combinated tests. Developed the new types of the test methods for increasing the accuracy of measurements of IMTS with non-linear TF, new types of combinated tests $kx \pm \theta$ were used in comparison with known methods based on additional measurements of the simple additive and multiplicative tests. When used n number of combined tests at values of parameters of MM of TF of initial MS different from zero MTE has degree (n-1) regarding to x. If during the using *m* number of the additive and combined tests at $n = 2m$ of values of parameters of MM of TF of initial MS that are not equal zero constant component $\theta_в$ fulfills condition $\theta_в - \theta_{в-1} = \theta_1, \quad (1 \le в \le m)$, then MTE has m number of multiplicative and combined tests regarding to *x*. At using the linear approximation by parts in the identification of the non-linear TF of initial MS uncorrelated components of errors of measuring results are reduced significantly.

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