

BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

M.Suganya^[1],A.Manonmani^[2]

^[1]Research Scholar, Department of Mathematics, LRG Government Arts college for Women,
Tirupur.

^[2]Assistant Professor, Department of Mathematics, LRG Government Arts college for Women,
Tirupur.

Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range [0,1] and [-1,0][21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X), then the necessity operator on A is defined as

$$i. \quad []A = \left\{ \left\langle x, \left[\begin{array}{l} \underline{\mu}^P(x), \overline{\mu}^P(x) \\ \underline{\nu}^{AL}(x), \overline{\nu}^{AU}(x) \end{array} \right], \left[\begin{array}{l} 1 - \underline{\mu}^P(x), 1 - \overline{\mu}^P(x) \\ \underline{\nu}^{AU}(x), \overline{\nu}^{AL}(x) \end{array} \right] \right\rangle \mid x \in X \right\}$$

2.1. Theorem:

Let (X, \mathcal{I}) be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X, we can also construct several BIVIFTSs on X as

$$\mathcal{I}_N = \{ []A | A \in \mathcal{I} \}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and \mathcal{I} be a family of bipolar interval valued intuitionistic fuzzy subset of S. The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if \mathcal{I} satisfies the following axioms

- i. $0, 1_s \in \mathcal{I}$
- ii. If $\{A_i; i \in I\} \subseteq \mathcal{I}$, then $\bigcap_{i=1}^{\infty} A_i \in \mathcal{I}$
- iii. If $A_1, A_2, A_3, \dots, A_n \in \mathcal{I}$, then $\bigcap_{i=1}^n A_i \in \mathcal{I}$

Let A_1, A_2, \dots, A_i be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

- i. obviously $0, 1_s \in \mathcal{I}_N$
- ii.

$$A \sqcap B = \left\{ \left[\begin{matrix} \underline{x} \underline{p}_{(A \sqcap B)L} (x), \underline{p}_{(A \sqcap B)U} (x) \\ \underline{x} \underline{p}_{(A \sqcap B)L} (x), \underline{p}_{(A \sqcap B)U} (x) \end{matrix} \right], \left[\begin{matrix} \underline{x} \underline{p}_{(A \sqcap B)L} (x), \underline{p}_{(A \sqcap B)U} (x) \\ \underline{x} \underline{p}_{(A \sqcap B)L} (x), \underline{p}_{(A \sqcap B)U} (x) \end{matrix} \right] \mid x \in X \right\}$$

where

$$\underline{p}_{(A \sqcap B)L} (x) = \min \{ \underline{p}_{AL} (x), \underline{p}_{BL}(x) \}$$

$$\begin{aligned} \mu_{(A \sqcup B)U}^P(x) &= \max \{ \mu_{AU}^P(x), \mu_{BU}^P(x) \} \\ \mu_{(A \sqcup B)L}^N(x) &= \max \{ \mu_{AL}^N(x), \mu_{BL}^N(x) \} \\ \mu_{(A \sqcup B)U}^N(x) &= \min \{ \mu_{AU}^N(x), \mu_{BU}^N(x) \} \\ \mu_{(A \sqcup B)L}^P(x) &= \min \{ \mu_{AL}^P(x), \mu_{BL}^P(x) \} \\ \mu_{(A \sqcap B)U}^P(x) &= \max \{ \mu_{AU}^P(x), \mu_{BU}^P(x) \} \\ \mu_{(A \sqcap B)L}^N(x) &= \max \{ \mu_{AL}^N(x), \mu_{BL}^N(x) \} \\ \mu_{(A \sqcap B)U}^N(x) &= \min \{ \mu_{AU}^N(x), \mu_{BU}^N(x) \} \end{aligned}$$

$$\mu_{[A]_1 \sqcap [A]_2} = \left\langle \begin{array}{l} \left[\mu_{[A_1 \sqcap A_2]L}^P(x), \mu_{[A_1 \sqcap A_2]U}^P(x) \right], \\ \left[\mu_{[A_1 \sqcap A_2]L}^N(x), \mu_{[A_1 \sqcap A_2]U}^N(x) \right], \\ \left[\mu_{[A_1 \sqcap A_2]L}^P(x), \mu_{[A_1 \sqcap A_2]U}^P(x) \right], \\ \left[\mu_{[A_1 \sqcap A_2]L}^N(x), \mu_{[A_1 \sqcap A_2]U}^N(x) \right] \end{array} \right\rangle | x \in X$$

where

$$\begin{aligned} \mu_{([A_1 \sqcap A_2]L)}^P(x) &= \min \{ \mu_{[A_1]L}^P(x), \mu_{[A_2]L}^P(x) \} \\ \mu_{([A_1 \sqcap A_2]U)}^P(x) &= \max \{ \mu_{[A_1]U}^P(x), \mu_{[A_2]U}^P(x) \} \\ \mu_{([A_1 \sqcap A_2]L)}^N(x) &= \max \{ \mu_{[A_1]L}^N(x), \mu_{[A_2]L}^N(x) \} \\ \mu_{([A_1 \sqcap A_2]U)}^N(x) &= \min \{ \mu_{[A_1]U}^N(x), \mu_{[A_2]U}^N(x) \} \\ \mu_{([A_1 \sqcup A_2]L)}^P(x) &= \min \{ \mu_{[A_1]L}^P(x), \mu_{[A_2]L}^P(x) \} \\ \mu_{([A_1 \sqcup A_2]U)}^P(x) &= \max \{ \mu_{[A_1]U}^P(x), \mu_{[A_2]U}^P(x) \} \\ \mu_{([A_1 \sqcup A_2]L)}^N(x) &= \max \{ \mu_{[A_1]L}^N(x), \mu_{[A_2]L}^N(x) \} \\ \mu_{([A_1 \sqcup A_2]U)}^N(x) &= \min \{ \mu_{[A_1]U}^N(x), \mu_{[A_2]U}^N(x) \} \end{aligned}$$

then

$$\begin{aligned} \mu_{([A_1 \sqcap A_2]L)}^P(x) &= \min \{ \mu_{A_1L}^P(x), \mu_{A_2L}^P(x) \} \\ \mu_{([A_1 \sqcap A_2]U)}^P(x) &= \max \{ \mu_{A_1U}^P(x), \mu_{A_2U}^P(x) \} \\ \mu_{([A_1 \sqcap A_2]L)}^N(x) &= \max \{ \mu_{A_1L}^N(x), \mu_{A_2L}^N(x) \} \end{aligned}$$

$$\begin{aligned} \mathbb{I}_{(A_1 \square A_2)U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \} \\ 1 \square \mathbb{I}_{(A_1 \square A_2)L}^P(x) &= \min \{ 1 \square \mathbb{I}_{A_1L}^P(x), 1 \square \mathbb{I}_{A_2L}^P(x) \} \\ 1 \square \mathbb{I}_{(A_1 \square A_2)U}^P(x) &= \max \{ 1 \square \mathbb{I}_{A_1U}^P(x), 1 \square \mathbb{I}_{A_2U}^P(x) \} \\ 1 \square \mathbb{I}_{(A_1 \square A_2)L}^N(x) &= \max \{ 1 \square \mathbb{I}_{A_1L}^N(x), 1 \square \mathbb{I}_{A_2L}^N(x) \} \\ 1 \square \mathbb{I}_{(A_1 \square A_2)U}^N(x) &= \min \{ 1 \square \mathbb{I}_{A_1U}^N(x), 1 \square \mathbb{I}_{A_2U}^N(x) \} \end{aligned}$$

$$\begin{aligned} \mathbb{I}_{A_1 \square A_2} &= \left\langle \begin{aligned} & \left[\mathbb{I}_{(A_1 \square A_2)L}^N(x), \mathbb{I}_{(A_1 \square A_2)U}^P(x) \right], \\ & \left[\mathbb{I}_{(A_1 \square A_2)L}^P(x), \mathbb{I}_{(A_1 \square A_2)U}^N(x) \right], \\ & \left[\mathbb{I}_{(A_1 \square A_2)L}^N(x), \mathbb{I}_{(A_1 \square A_2)U}^N(x) \right], \\ & \left[\mathbb{I}_{(A_1 \square A_2)L}^P(x), \mathbb{I}_{(A_1 \square A_2)U}^P(x) \right] \end{aligned} \right\rangle | x \square X \\ \mathbb{I}_{A_1 \square A_2 \square \dots \square A_i} &= \left\langle \begin{aligned} & \left[\mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^N(x), \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^P(x) \right], \\ & \left[\mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^P(x), \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^N(x) \right], \\ & \left[\mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^N(x), \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^N(x) \right], \\ & \left[\mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^P(x), \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^P(x) \right] \end{aligned} \right\rangle | x \square X \end{aligned}$$

where

$$\begin{aligned} \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^P(x) &= \min \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^P(x) &= \max \{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^N(x) &= \max \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^P(x) &= \min \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^P(x) &= \max \{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^N(x) &= \max \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \} \\ \mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \} \end{aligned}$$

then

$$\mathbb{I}_{(A_1 \square A_2 \square \dots \square A_i)L}^P(x) = \min \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \}$$

$$\begin{aligned}
 \mathbb{I}^P_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_U])} (x) &= \max \{ \mathbb{I}^P_{A_1U}(x), \mathbb{I}^P_{A_2U}(x), \dots, \mathbb{I}^P_{A_iU}(x) \} \\
 \mathbb{I}^N_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x) &= \max \{ \mathbb{I}^N_{A_1L}(x), \mathbb{I}^N_{A_2L}(x), \dots, \mathbb{I}^N_{A_iL}(x) \} \\
 \mathbb{I}^N_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x) &= \min \{ \mathbb{I}^N_{A_1U}(x), \mathbb{I}^N_{A_2U}(x), \dots, \mathbb{I}^N_{A_iU}(x) \} \\
 1 \sqcap \mathbb{I}^P_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x) &= \min \{ 1 \sqcap \mathbb{I}^P_{A_1L}(x), 1 \sqcap \mathbb{I}^P_{A_2L}(x), \dots, 1 \sqcap \mathbb{I}^P_{A_iL}(x) \} \\
 1 \sqcap \mathbb{I}^P_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x) &= \max \{ 1 \sqcap \mathbb{I}^P_{A_1U}(x), 1 \sqcap \mathbb{I}^P_{A_2U}(x), \dots, 1 \sqcap \mathbb{I}^P_{A_iU}(x) \} \\
 1 \sqcap \mathbb{I}^N_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x) &= \max \{ 1 \sqcap \mathbb{I}^N_{A_1L}(x), 1 \sqcap \mathbb{I}^N_{A_2L}(x), \dots, 1 \sqcap \mathbb{I}^N_{A_iL}(x) \} \\
 1 \sqcap \mathbb{I}^N_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x) &= \min \{ 1 \sqcap \mathbb{I}^N_{A_1U}(x), 1 \sqcap \mathbb{I}^N_{A_2U}(x), \dots, 1 \sqcap \mathbb{I}^N_{A_iU}(x) \}
 \end{aligned}$$

$$\left[\bigcap_{i=1}^n A_i \right] = \left\langle \begin{array}{l} x, \mathbb{I}^P_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x), \mathbb{I}^P_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x), \\ \mathbb{I}^N_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x), \mathbb{I}^N_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x), \\ 1 \sqcap \mathbb{I}^P_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x), 1 \sqcap \mathbb{I}^P_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x), \\ 1 \sqcap \mathbb{I}^N_{([A_1 \sqcap [A_2 \sqcap \dots [A_i]_L])} (x), 1 \sqcap \mathbb{I}^N_{([A_1 \sqcup [A_2 \sqcup \dots [A_i]_U])} (x) \end{array} \right\rangle | x \in X$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

iii.

$$A \sqcap B = \left\langle \left[x, \mathbb{I}^P_{(A \sqcap B)L}(x), \mathbb{I}^P_{(A \sqcap B)U}(x) \right], \left[\mathbb{I}^N_{(A \sqcap B)L}(x), \mathbb{I}^N_{(A \sqcap B)U}(x) \right] \right\rangle | x \in X$$

where

$$\begin{aligned}
 \mathbb{I}^P_{(A \sqcap B)L}(x) &= \max \{ \mathbb{I}^P_{AL}(x), \mathbb{I}^P_{BL}(x) \} \\
 \mathbb{I}^P_{(A \sqcap B)U}(x) &= \min \{ \mathbb{I}^P_{AU}(x), \mathbb{I}^P_{BU}(x) \} \\
 \mathbb{I}^N_{(A \sqcap B)L}(x) &= \min \{ \mathbb{I}^N_{AL}(x), \mathbb{I}^N_{BL}(x) \} \\
 \mathbb{I}^N_{(A \sqcap B)U}(x) &= \max \{ \mathbb{I}^N_{AU}(x), \mathbb{I}^N_{BU}(x) \} \\
 \mathbb{I}^P_{(A \sqcup B)L}(x) &= \max \{ \mathbb{I}^P_{AL}(x), \mathbb{I}^P_{BL}(x) \} \\
 \mathbb{I}^P_{(A \sqcup B)U}(x) &= \min \{ \mathbb{I}^P_{AU}(x), \mathbb{I}^P_{BU}(x) \} \\
 \mathbb{I}^N_{(A \sqcup B)L}(x) &= \min \{ \mathbb{I}^N_{AL}(x), \mathbb{I}^N_{BL}(x) \}
 \end{aligned}$$

$$\mu_{(A \sqcup B)U}^N(x) = \max \{ \mu_{AU}^N(x), \mu_{BU}^N(x) \}$$

then

$$(\mu_{A_1} \sqcup \mu_{A_2})(x) = \begin{cases} \left[\begin{array}{l} \mu_{(A_1 \sqcup A_2)L}^L(x), \mu_{(A_1 \sqcup A_2)U}^L(x) \\ \mu_{(A_1 \sqcup A_2)L}^N(x), \mu_{(A_1 \sqcup A_2)U}^N(x) \\ \mu_{(A_1 \sqcup A_2)L}^P(x), \mu_{(A_1 \sqcup A_2)U}^P(x) \\ \mu_{(A_1 \sqcup A_2)L}^Q(x), \mu_{(A_1 \sqcup A_2)U}^Q(x) \end{array} \right] \end{cases} | x \in X$$

where

$$\begin{aligned} \mu_{(A_1 \sqcup A_2)L}^P(x) &= \max \{ \mu_{A_1L}^P(x), \mu_{A_2L}^P(x) \} \\ \mu_{(A_1 \sqcup A_2)U}^P(x) &= \min \{ \mu_{A_1U}^P(x), \mu_{A_2U}^P(x) \} \\ \mu_{(A_1 \sqcup A_2)L}^N(x) &= \min \{ \mu_{A_1L}^N(x), \mu_{A_2L}^N(x) \} \\ \mu_{(A_1 \sqcup A_2)U}^N(x) &= \max \{ \mu_{A_1U}^N(x), \mu_{A_2U}^N(x) \} \\ \mu_{(A_1 \sqcup A_2)L}^Q(x) &= \max \{ \mu_{A_1L}^Q(x), \mu_{A_2L}^Q(x) \} \\ \mu_{(A_1 \sqcup A_2)U}^Q(x) &= \min \{ \mu_{A_1U}^Q(x), \mu_{A_2U}^Q(x) \} \\ \mu_{(A_1 \sqcup A_2)L}^N(x) &= \min \{ \mu_{A_1L}^N(x), \mu_{A_2L}^N(x) \} \\ \mu_{(A_1 \sqcup A_2)U}^N(x) &= \max \{ \mu_{A_1U}^N(x), \mu_{A_2U}^N(x) \} \end{aligned}$$

then

$$\begin{aligned} \mu_{(A_1 \sqcup A_2)L}^P(x) &= \max \{ \mu_{A_1L}^P(x), \mu_{A_2L}^P(x) \} \\ \mu_{(A_1 \sqcup A_2)U}^P(x) &= \min \{ \mu_{A_1U}^P(x), \mu_{A_2U}^P(x) \} \\ \mu_{(A_1 \sqcup A_2)L}^N(x) &= \min \{ \mu_{A_1L}^N(x), \mu_{A_2L}^N(x) \} \\ \mu_{(A_1 \sqcup A_2)U}^N(x) &= \max \{ \mu_{A_1U}^N(x), \mu_{A_2U}^N(x) \} \\ 1 \sqcup \mu_{(A_1 \sqcup A_2)L}^P(x) &= \max \{ 1 \sqcup \mu_{A_1L}^P(x), 1 \sqcup \mu_{A_2L}^P(x) \} \\ 1 \sqcup \mu_{(A_1 \sqcup A_2)U}^P(x) &= \min \{ 1 \sqcup \mu_{A_1U}^P(x), 1 \sqcup \mu_{A_2U}^P(x) \} \\ 1 \sqcup \mu_{(A_1 \sqcup A_2)L}^N(x) &= \min \{ 1 \sqcup \mu_{A_1L}^N(x), 1 \sqcup \mu_{A_2L}^N(x) \} \end{aligned}$$

$$\begin{aligned}
 & 1 \square \square^N_{([]A_1 \square []A_2)U} (x) = \max \{ 1 \square \square^N_{A_1U} (x), 1 \square \square^N_{A_2U} (x) \} \\
 & \square []A_1 \square []A_2 = \left(\begin{array}{l} x, \square \square^P_{([]A_1 \square []A_2)L} (x), \square \square^P_{([]A_1 \square []A_2)U} (x) \square, \\ \square \square^N_{([]A_1 \square []A_2)L} (x), \square \square^N_{([]A_1 \square []A_2)U} (x) \square, \\ 1 \square \square^P_{([]A_1 \square []A_2)L} (x), 1 \square \square^P_{([]A_1 \square []A_2)U} (x) \square, \\ 1 \square \square^N_{([]A_1 \square []A_2)L} (x), 1 \square \square^N_{([]A_1 \square []A_2)U} (x) \square \end{array} \right) | x \square X \square \square^N
 \end{aligned}$$

$$\begin{aligned}
 & \square []A_1 \square []A_2 \square \dots \square []A_i = \left(\begin{array}{l} x, \square \square^P_{([]A_1 \square []A_2 \square \dots []A_i)L} (x), \square \square^P_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) \square, \\ \square \square^N_{([]A_1 \square []A_2 \square \dots []A_i)L} (x), \square \square^N_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) \square, \\ \square \square^P_{([]A_1 \square []A_2 \square \dots []A_i)L} (x), \square \square^P_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) \square, \\ \square \square^N_{([]A_1 \square []A_2 \square \dots []A_i)L} (x), \square \square^N_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) \square \end{array} \right) | x \square X \square \square^N
 \end{aligned}$$

where

$$\square^P_{([]A_1 \square []A_2 \square \dots []A_i)L} (x) = \max \{ \square^P_{[]A_1L} (x), \square^P_{[]A_2L} (x), \dots, \square^P_{[]A_iL} (x) \}$$

$$\square^P_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) = \min \{ \square^P_{[]A_1U} (x), \square^P_{[]A_2U} (x), \dots, \square^P_{[]A_iU} (x) \}$$

$$\square^N_{([]A_1 \square []A_2 \square \dots []A_i)L} (x) = \min \{ \square^N_{[]A_1L} (x), \square^N_{[]A_2L} (x), \dots, \square^N_{[]A_iL} (x) \}$$

$$\square^N_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) = \max \{ \square^N_{[]A_1U} (x), \square^N_{[]A_2U} (x), \dots, \square^N_{[]A_iU} (x) \}$$

$$\square^P_{([]A_1 \square []A_2 \square \dots []A_i)L} (x) = \max \{ \square^P_{[]A_1L} (x), \square^P_{[]A_2L} (x), \dots, \square^P_{[]A_iL} (x) \}$$

$$\square^P_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) = \min \{ \square^P_{[]A_1U} (x), \square^P_{[]A_2U} (x), \dots, \square^P_{[]A_iU} (x) \}$$

$$\square^N_{([]A_1 \square []A_2 \square \dots []A_i)L} (x) = \min \{ \square^N_{[]A_1L} (x), \square^N_{[]A_2L} (x), \dots, \square^N_{[]A_iL} (x) \}$$

$$\square^N_{([]A_1 \square []A_2 \square \dots []A_i)U} (x) = \max \{ \square^N_{[]A_1U} (x), \square^N_{[]A_2U} (x), \dots, \square^N_{[]A_iU} (x) \}$$

then

$$\square^P_{([]A_1 \square []A_2 \square \dots []A_i)L} (x) = \max \{ \square^P_{A_1L} (x), \square^P_{A_2L} (x), \dots, \square^P_{A_iL} (x) \}$$

$$\begin{aligned}
 \bigcap_{i=1}^n A_i &= \left\{ x \in X \mid \begin{aligned} & \bigcap_{i=1}^n \left(\left(\bigcap_{j=1}^n A_{ij} \right)^U \right) \cap \left(\bigcap_{j=1}^n A_{ij} \right)^L \\ & \bigcap_{i=1}^n \left(\left(\bigcap_{j=1}^n A_{ij} \right)^L \right) \cap \left(\bigcap_{j=1}^n A_{ij} \right)^U \\ & \bigcap_{i=1}^n \left(\left(\bigcap_{j=1}^n A_{ij} \right)^U \right) \cap \left(\bigcap_{j=1}^n A_{ij} \right)^L \\ & \bigcap_{i=1}^n \left(\left(\bigcap_{j=1}^n A_{ij} \right)^L \right) \cap \left(\bigcap_{j=1}^n A_{ij} \right)^U \end{aligned} \right\}
 \end{aligned}$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

References:

[1] Andrijevic.D, “Semipreopen sets”, Mat.Vesnic, 38 (1986), 24 -32.
 [2] Azhagappan. M and M.Kamaraj, “Notes on bipolar valued fuzzy rw-closed and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces”, International Journal of Mathematical Archive, 7(3), 30-36, 2016.
 [3] Bhattacharya.B., and Lahiri. B.K., “Semi-generalized closed set in topology”, Indian Jour.Math., 29 (1987), 375 -382.
 [4] Chang.C.L., “Fuzzy topological spaces”, JI. Math. Anal. Appl., 24(1968), 182 -190.
 [5] Dontchev.J., “On generalizing semipreopen sets”, Mem. Fac. sci. Kochi. Univ. Ser. A, Math.,16 (1995), 35 - 48.

- [6] Ganguly.S and Saha.S, “A note on fuzzy Semipreopen sets in fuzzy topological spaces”, Fuzzy sets and system, 18 (1986), 83 - 96.
- [7] Indira.R, Arjunan.K and Palaniappan.N, “Notes on interval valued fuzzy rw-closed, interval valued fuzzy rw-open sets in interval valued fuzzy topological space”, International Journal of Computational and Applied Mathematics.,Vol.3, No.1(2013), 23 -38.
- [8] Jeyabalan.R and K. Arjunan, “Notes on interval valued fuzzy generalized semipreclosed sets”, International Journal of Fuzzy Mathematics and Systems, Volume 3, Number 3 (2013), 215 -224.
- [9] Jeyabalan,R and Arjunan.K, “Interval valued fuzzy generalized semi-preclosed mappings”, IOSR Journal of Mathematics, Vol.8, Issue 5(2013), 40 – 47.
- [10] Lee, K.M., “Bipolar-valued fuzzy sets and their operations”, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, (2000), 307 -312.
- [11] Levine.N, “Generalized closed sets in topology”, Rend. Circ. Math. Palermo, 19 (1970),89 -96.
- [12] Mondal.T.K., “Topology of interval valued fuzzy sets”, Indian J. Pure Appl.Math. 30, No.1 (1999), 23 - 38.
- [13] Murugan.V, U.Karuppiah & M.Marudai, Notes on multi fuzzy rw-closed, multi fuzzy rw-open sets in multi fuzzy topological spaces, Bulletin of Mathematics and Statistics Research, Vo.4, Issue. 1, 2016, 174-179.
- [14] Sabu Sebastian, T.V.Ramakrishnan, Multi fuzzy sets, International Mathematical Forum, 5, no.50 (2010), 2471-2476.
- [15] Saraf.R.K and Khanna.K., “Fuzzy generalized semipreclosed sets”, Jour.Tripura. Math.Soc., 3 (2001), 59 - 68.
- [16] Selvam.R, K.Arjunan & KR.Balasubramanian,”Bipolar interval valued multi fuzzy generalized semipreclosed sets”, JASC Journal, Vol.6, ISS. 5(2019), 2688 -2697.
- [17] Vinoth.S & K.Arjunan, “A study on interval valued intuitionistic fuzzy generalized semipreclosed sets”, International Journal of Fuzzy Mathematics and Systems, Vol. 5, No. 1 (2015), 47 -55.
- [18] Vinoth.S & K.Arjunan, “Interval valued intuitionistic fuzzy generalized semipreclosed mappings”, International Journal of Mathematical Archive, 6(8) (2015), 129 -135.
- [19] Zadeh.L.A., “Fuzzy sets”, Information and control, Vol.8 (1965), 338 -353.

[20] K., Mohana; Princy R; and F. Smarandache. "An Introduction to Neutrosophic Bipolar Vague Topological Spaces." Neutrosophic Sets and Systems 29, 1 (2020).

[21] R.Selvam, K. Arjunan And Kr.Balasubramanian "Bipolar Interval Valued Multi Fuzzy Generalized Semipre Continuous Mappings" Volume: 06 Issue: 12 | Dec 2019.